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Some results on the 2-absorbing primary hyperideals in multiplicative hyperrings

Mohammad Ali Dehghanizadeh [1]
[1] Department of Basic Sciences, Technical and Vocational University(TVU), Tehran, Iran

Abstract. The structural investigation of regular prime ideals in the setting of multiplicative hyperrings constitutes a profound and evolving area within hyperstructure theory. This domain not only enhances the theoretical foundation of hyperalgebra but also provides a versatile framework for interpreting and generalizing classical algebraic concepts. By relaxing the conventional binary operations into hyperoperations, multiplicative hyperrings offer a rich algebraic environment where classical notions such as ideals, prime ideals, and their generalizations acquire novel and meaningful forms. In particular, the study of generalized prime-like objects-such as 2-absorbing δ-prime hyperideals-plays a central role in deepening our comprehension of the internal behavior of hyperrings. These generalized structures extend the traditional notion of prime ideals and allow us to explore more intricate algebraic phenomena that are otherwise obscured in the classical setting. The purpose of this paper is to rigorously explore and characterize the properties of 2-absorbing δ-prime hyperideals in multiplicative hyperrings. We aim to provide a comprehensive analysis of their defining conditions, structural implications, and potential interactions with other classes of hyperideals.

[1]Corresponding author: Mdehghanizadeh@tvu.ac.ir
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1. Introduction

The concept of hyperstructures was first introduced by F. Marty during the 8th Scandinavian Mathematics Congress in 1934, where he defined hypergroups [16]. A hypergroup, as articulated by Marty, is constructed from the Cartesian product of a non-empty set H along with a function that maps to the power set of H. In the subsequent decade of the 1940s, significant advancements in the theory of hyperstructures were made in various countries. In France, notable mathematicians such as F. Marty, M. Krasner, M. Kuntzmann, and R. Croisot contributed to the foundational results and implications of this theory. Meanwhile, in the United States, researchers like M. Dresher, O. Ore, W. Prenowitz, and H. Campaigne were also exploring similar concepts, while in Russia, A. Dietzman and A. Vikhrov, along with G. Zappa in Italy, furthered the study of hyperstructures [2, 3, 9, 10, 15, 18]. During the 1950s and 1960s, the focus shifted to semiregular hypergroups and hyperlattices, with A. Orsatti in Italy and M. Benado in Romania leading the research efforts. The year 1956 marked a significant milestone when Marc Krasner introduced the notions of hyperrings and hyperfields [2, 14]. The 1970s saw further exploration of subhypergroups and their interrelations with hyperstructures, particularly by M. Krasner, M. Koskas, and Y. Sureau in France. In Greece, J. Mittas and his students concentrated on canonical hypergroups, hyperrings, and hyperlattices [13, 17, 21]. The 1980s were characterized by the definition of various types of hyperrings, including Multiplicative hyperrings and General hyperrings, with the former being first identified by Rosaria Rota in 1982 [20]. The 2000s witnessed a resurgence of interest in hyperrings, with scholars such as B. Davvaz, Salasi, Asokkumar, Procesi, Kemprasit, and Velrajan contributing to the literature and expanding the field [6, 7, 12, 19, 22]. In classical algebra, the notion of δ-Prime ideals was established by Zhao Dongsheng in 2000, followed by the definition of 2-absorbing ideals by A. Badawi in 2007 [1]. The concept of regular prime ideals was introduced by J. A. Cox and A. J. Hetzel in 2008 [4]. More recently, in 2017, Zhao Dongsheng [23] and Brahim Fahid defined 2-absorbing δ-prime ideals in classical algebra, elucidating their various properties [11]. Hyperrings can be categorized into three types: a general hyperring is defined when both "+" and "." are hyperoperations; a Krasner hyperring occurs when the "+" operation is a normal operation, while a multiplicative hyperring is characterized by the "+" operation being a hyperoperation [5]. In this paper, we will focus on the investigation of 2-absorbing δ-prime hyperideals, exploring their properties and significance within the broader context of hyperstructures, and we complete the results obtained from Dehghanizadeh in [8].

2. Hyperideal Extension Function

From this paper onwards, R will be considered as a multiplicative hyperring.

Definition 2.1. Let R be a multiplicative hyperring. The function δ that takes all hyperideals of R to other hyperideals on the same hyperring is called a hyperideal extension function if the following conditions are satisfied:

(i) For each hyperideal I of R; $I \subseteq \delta(I)$,
(ii) For hyperideals P and Q of R; $P \subseteq Q$, while $\delta(P) \subseteq \delta(Q)$.

Example 2.2. The identity function $\delta_0(I) = I$ and $\delta_1(I) = \sqrt{I}$ are hyperideal extension functions.

Definition 2.3. Let R be a multiplicative hyperring. Let I be a hyperideal of R. Let δ be an extension function. For each $a, b \in R$; $ab \subseteq I$ and $a \notin I$, while $b \in \delta(I)$, then I is called a δ-prime hyperideal.

Example 2.4. Let us define the hyperproduct for the ring of integers $(\mathbb{Z}, +, .)$ as $x \circ y = \{2xy, 3xy\}$. In this case $(\mathbb{Z}, +, \circ)$ is a multiplicative hyperring. Let δ_0 be the extension function. In this hyperring, the hyperideal $I = 5\mathbb{Z}$ is a δ_0-prime hyperideal.

Definition 2.5. Let R be a multiplicative hyperring and I be a proper hyperideal of R. For each $a, b, c \in R$; If $abc \subseteq I$ while $ab \subseteq I$ or $bc \subseteq \delta(I)$ or $ac \subseteq \delta(I)$ then I is called a 2-absorbing δ-prime hyperideal of R.

Example 2.6. Let R be a hyperring and I a hyperideal of R. The hyperideal I is a 2-absorbing δ_1-prime hyperideal if and only if it is a 2-absorbing prime hyperideal. If I be a 2-absorbing hyperideal, then for all $a, b, c \in R$; such that $abc \subseteq I$ we must have $ab \subseteq I$ or $bc \subseteq I = \delta(I)$ or $ac \subseteq I = \delta_0(I)$, then I is a 2-absorbing δ_0-prime hyperideal. Similarly, the other side is shown.

Example 2.7. Let R be a multiplicative hyperring and I a hyperideal of R. I, 2-absorbing δ_1-is a prime hyperideal if and only if I is a 2-absorbing prime hyperideal. Since I be a 2-absorbing δ_1-primer hyperideal. For all $a, b, c \in R$; $abc \subseteq I$ while $ab \subseteq I$ or $bc \subseteq \sqrt{I} = \delta_1(I)$ or $ac \subseteq \sqrt{I} = \delta_1(I)$, then the 2-absorbing prime hyperideal is found. Similarly, the converse can be shown.

For every δ-extension function, every 2-absorbing hyperideal in a multiplicative hyperring is a 2-absorbing δ-prime hyperideal. But the converse is not true.

Example 2.8. Consider the ring of integers $(\mathbb{Z}, +, .)$ and define the hyperproduct as $x \circ y = \{2xy, 3xy\}$. In this case $(\mathbb{Z}, +, \circ)$ is a multiplicative hyperring. Let δ_1 be the extension function. In this hyperrring, the hyperideal $I = 12\mathbb{Z}$ is a 2-absorbing δ_1-prime hyperideal. But it is not a 2-absorbing hyperideal. Let $a = 2, b = 2, c = 3$. $a \circ b \circ c = \{48, 72, 108\} \subseteq I = 12\mathbb{Z}$ while $a \circ b = \{8, 12\} \nsubseteq I$ and $b \circ c = \{12, 18\} \nsubseteq I$ and $a \circ c = \{12, 18\} \nsubseteq I$, so it is not a 2-absorbing hyperideal. But for the extension function δ_1; Since $\sqrt{I} = 6\mathbb{Z}$, at least one of the products falls into the radical.

A 2-absorbing δ-prime hyperideal example;

Example 2.9. Consider the ring of integers $(\mathbb{Z}, +, .)$. Let's define the hyperproduct as $x \circ y = \{5xy, 6xy\}$. In this case $(\mathbb{Z}, +, \circ)$ is a multiplicative hyperring. Let's consider the hyperideal $I = 30\mathbb{Z}$. Let the extension function be a function that takes the hyperideal I to the radical of I. The radical of the hyperideal I is $a^2 = \{5a^2, 6a^2\}$,
$a^3 = \{25a^3, 30a^3, 36a^3\}, a^4 = \{125a^4, 150a^4, 180a^4, 150a^4, 216a^4\}$,
$..., a^n = \{5^{n-1}, a^n, 6.5^{n-2}.a^n, ...\}$. $\sqrt{I} = 30\mathbb{Z}$ is. For $a = 2, b = 3, c = 5; 2 \circ 3 \circ 5 = \{750, 900, 1080\} \subseteq I$ while $2 \circ 3 = \{30, 36\} \nsubseteq I$ and $3 \circ 5 = \{75, 90\} \nsubseteq \delta_1(I) = 30\mathbb{Z}$ and $2 \circ 5 = \{50, 60\} \nsubseteq \delta_1(I) = 30\mathbb{Z}$, so I is not a 2-absorbing δ-prime hyperideal.

3. 2-absorbing δ-prime hyperideals

In this section, we study 2-absorbing δ-prime hyperideals.

Theorem 3.1. *Let δ and γ be extension functions defined on the family of hyperideals of a hyperring R such that $\delta(I) \subseteq \gamma(I)$ for every hyperideal I of R. If I is a 2-absorbing δ-prime hyperideal of R, then I is also a 2-absorbing γ-prime hyperideal of R.*

Proof. Since $\delta(I) \subseteq \gamma(I)$ for every hyperideal I, it follows that if $ab \subseteq \gamma(I)$ for some $a, b \in R$, then necessarily $ab \subseteq \delta(I)$, because $\delta(I)$ is contained in $\gamma(I)$. Now assume that I is a 2-absorbing δ-prime hyperideal of R. By definition, whenever $a, b, c \in R$ satisfy $abc \subseteq \delta(I)$, we must have either $ab \subseteq I$, $ac \subseteq I$, or $bc \subseteq I$. Consider any $a, b, c \in R$ such that $abc \subseteq \gamma(I)$. Since $\delta(I) \subseteq \gamma(I)$, we have $abc \subseteq \gamma(I)$ implies $abc \subseteq \delta(I)$. Applying the δ-prime assumption on I, we obtain $ab \subseteq I$ or $ac \subseteq I$ or $bc \subseteq I$. Therefore, the same condition holds for elements whose product lies in $\gamma(I)$, showing that I satisfies the definition of a 2-absorbing

γ-prime hyperideal. Hence, every 2-absorbing δ-prime hyperideal is also a 2-absorbing γ-prime hyperideal, as required. □

This in the theorem suggests that if a hyperideal I is 2-absorbing and also a 2-absorbing δ-prime hyperideal (where δ is a specific extension function), then it automatically follows that I is a 2-absorbing γ-prime hyperideal (assuming γ is another extension function satisfying certain conditions). In simpler terms, this means that if a hyperideal I can be extended in a certain way and also maintains specific properties related to the extension functions δ and γ, then it will also possess those properties when related to the other extension function γ. This result provides a connection between different extension functions and reinforces the importance of understanding the behavior of hyperideals within multiplicative hyperrings. It also highlights the significance of hyperideals having both the 2-absorbing and 2-absorbing δ-prime properties, as they automatically inherit the 2-absorbing γ-prime property.

Remark 3.2. The intersection of two 2-absorbing δ-prime hyperideals is not a 2-absorbing δ-prime hyperideal in general.

Proof. Let I and J be 2-absorbing δ-prime hyperideals. Let $K = I \cap J$. For $\forall a, b, c \in R$; let $abc \subseteq K$ and $ab \nsubseteq K$. Then $ab \nsubseteq K$ are $ab \nsubseteq I$ and $ab \nsubseteq J$. Let I and J be 2-absorbing is a prime hyperideal if $ab \nsubseteq I$ then $ac\delta(I)$ or $bc\delta(I)$ and if $ab \nsubseteq J$ then $ac\delta(J)$ or $bc\delta(J)$.

If $ac\delta(I)$ and $bc\delta(J)$ then $ac \nsubseteq (K)$ and $bc \nsubseteq (K)$ are found. □

Theorem 3.3. *Let R be a hyperring, a hyperideal extension function, and P_1 and P_2 are prime hyperideals. Then $P_1 \cap P_2$ is a 2-absorbing prime hyperideal of the multiplicative hyperring R.*

Proof. For all $a, b, c \in R$; let $abc \subseteq (P_1 \cap P_2)$ and $ab \nsubseteq P_1 \cap P_2$. We need to show that $bc \subseteq \delta(P_1 \cap P_2)$ or $ac \subseteq (P_1 \cap P_2)$. If $abc \subseteq (P_1 \cap P_2)$, then $abc \subseteq P_1)$ and $abc \subseteq P_2$. Since $ab \nsubseteq P_1 \cap P_2$ then $ab \nsubseteq P_1$ and $ab \nsubseteq P_2$. Since P_1 and P_2 are prime hyperideals, then $c \in P_1$ when $abc \subseteq P_1$ and $ab \nsubseteq P_1$. If $abc \subseteq P_2$ and $ab \nsubseteq P_2$ then $c \in P_2$. If $c \in P_1 \cap P_2$ then $bc \subseteq (P_1 \cap P_2)$ and $ac \subseteq (P_1 \cap P_2)$ are obtained. From the definition of $P_1 \cap P_2 \subseteq \delta(P_1 \cap P_2)$ then $bc \subseteq \delta(P_1 \cap P_2)$ and $ac \subseteq \delta(P_1 \cap P_2)$, and hence $P_1 \cap P_2$ is a 2-containing prime hyperideal. □

Example 3.4. Consider the set $A = \{2, 7\}$ in the ring of integers $(\mathbb{Z}, +, .)$. $(\mathbb{Z}, +, \circ)$ is a multiplicative hyperring with the product defined as $x \circ y = \{2xy, 7xy\}$, $P_1 = (2)$ and $P_2 = (3)$ are prime hyperideals. $P_1 \cap P_2 = (2) \cap (3) = (6)$ is a 2-absorbing prime hyperideal.

Theorem 3.5. *Let R be a multiplicative hyperring and P be a 2-absorbing prime hyperideal of R. For any $a, b \in R$; then $abI \subseteq P$ and $ab \nsubseteq P$, then $aI \subseteq \delta(P)$ or $bI \subseteq \delta(P)$.*

Proof. For some $a, b \in R$; Let $abI \subseteq P$ and $ab \nsubseteq P$. Let $aI \nsubseteq \delta(P)$ and $bI \nsubseteq \delta(P)$ be assumed. For $c, d \in I$; we obtain $ac \nsubseteq \delta(P)$ and $bd \nsubseteq \delta(P)$. Since P is a 2-containing prime hyperideal, when $abc \subseteq P$ and $ab \nsubseteq P$, $ac \nsubseteq \delta(P)$ is $bc \subseteq \delta(P)$ and when $abd \subseteq P$ and $ab \nsubseteq P$, $bd \nsubseteq \delta(P)$ is bd ensures that $ad \subseteq \delta(P)$. Since $abc + abd = ab(c+d) \subseteq P$ and $ab \nsubseteq P$, either $a(c+d) \subseteq \delta(P)$ or $b(c+d) \subseteq \delta(P)$ is obtained. If $a(c+d) \subseteq \delta(P)$ then since $ad \subseteq \delta(P)$ then the contradiction $ac \subseteq \delta(P)$ is obtained.

If $b(c+d) subseteqdelta(P)$, then since $bc \subseteq \delta(P)$, so the contradiction $bd \subseteq \delta(P)$ is obtained. From here we get the result $aI \subseteq \delta(P)$ or $bI \subseteq \delta(P)$.

□

Example 3.6. Consider the set $A = \{2, 5\}$ in the ring of integers $(\mathbb{Z}, +, .)$. $x \circ y = \{2xy, 5xy\}$together with the product we define as $(\mathbb{Z}, +, \circ)$ is a multiplicative hyperring. $P = 3\mathbb{Z}$ is a 2-containing prime hyperideal. Let $a = 4, b = 5$ and $I = 6\mathbb{Z}$. $a \circ b = \{40, 100\} \nsubseteq P$.
$a \circ b \circ I = \{480, 1200, 960, 2400, 1440, 3600, 1200, 3000, 2400, 6000, 3600, 1200, 3000, 2400, 6000, 3600, 9000, P = 3\mathbb{Z}\}$ while $a \circ I = 4I = \{48, 120, 144, 360, ...\} \subseteq \delta_0(P)$, $\delta_1(P)$ is found.

Theorem 3.7. *Let R be a multiplicative hyperring and P a proper hyperideal of R. P is 2-containing δ-prime if and only if for some hyperideals I, J, K of R if $IJK \subseteq P$, then $IJ \subseteq P$ or $IKsubseteq\delta(P)$ or $JK \subseteq \delta(P)$.*

Proof. Let P be a 2-containing-prime hyperideal. Let $IJK \subseteq P$ and $IJ \nsubseteq P$. We must show that $IK \subseteq \delta(P)$ or $JK \subseteq \delta(P)$. Suppose there are $IK \nsubseteq \delta(P)$ and $JK \nsubseteq \delta(P)$. There exists $\exists r \in I, s \in J$ such that $rK \nsubseteq \delta(P)$ and $sK \nsubseteq \delta(P)$. Since $rsK \subseteq P, rK \nsubseteq \delta(P)$, $sK \nsubseteq \delta(P)$, we obtain $rs \subseteq P$ from the Theorem 3.5. Since $IJ \nsubseteq P$, there exist $a \in I$ and $b \in J$ such that $ab \nsubseteq P$. Since $abK \subset P$ and $ab \nsubseteq P$, then $aK \subseteq \delta(P)$ or $bK \subseteq \delta(P)$. Let us assume that $aK \subseteq \delta(P)$ and $bK \nsubseteq \delta(P)$. Since $rbK \subseteq P, bK \nsubseteq \delta(P)$ and $rK \nsubseteq \delta(P)$, then $rb \subseteq P$. Since $(a+r)bK \subseteq P, aK \subseteq \delta(P)$ and $rK \nsubseteq \delta(P)$, then $(a+r)K \nsubseteq \delta(P)$. If $(a+r)K \subseteq \delta(P)$ then $(a+r)k \subseteq \delta(P)$ and $rk \nsubseteq \delta(P)$ are obtained. Since $aK \subseteq \delta(P)$ and $rksubseteq\delta(P)$, by the Theorem 3.5 $(a+r)b \subseteq P$ is obtained. Since $rb \subseteq P$, $ab \subseteq P$. Thus, a contradiction is obtained. Assuming $aK \nsubseteq \delta(P)$ and $bKsubseteq\delta(P)$, we obtain a contradiction in a similar way. □

4. Some Application

In this section, we illustrate the theorem 3.1 through a few examples showing how a 2-absorbing δ-prime hyperideal automatically becomes a

2-absorbing γ-prime hyperideal whenever $\delta(I) \subseteq \gamma(I)$ holds. For simplicity, we use classical commutative rings equipped with natural hyperideal structures.

(1) Extension functions on the ring of integers:

Example 4.1. Let $R = \mathbb{Z}$, the ring of integers. Define two extension functions as follows:

$$\delta(I) = 2I = \{2x \mid x \in I\}, \qquad \gamma(I) = I.$$

Then clearly $\delta(I) \subseteq \gamma(I)$ for every ideal I.

Let $I = 6\mathbb{Z}$. We claim that I is a 2-absorbing δ-prime hyperideal. Suppose $a, b, c \in \mathbb{Z}$ with $abc \in \delta(I) = 12\mathbb{Z}$. Then $abc = 12k$ for some $k \in \mathbb{Z}$. Hence at least one of ab, ac, or bc is divisible by 6, that is, $ab \in I$ or $ac \in I$ or $bc \in I$.

Since $\delta(I) \subseteq \gamma(I)$, whenever $abc \in \gamma(I) = 6\mathbb{Z}$, we also have $abc \in \delta(I)$. By the theorem 3.1, I must also be a 2-absorbing γ-prime hyperideal.

$6\mathbb{Z}$ is 2-absorbing γ-prime because it was 2-absorbing δ-prime under $\delta(I) = 2I$ and $\gamma(I) = I$.

(2) Extensions on a quotient ring:

Example 4.2. Let $R = \mathbb{Z}_{12}$, and consider the hyperideal $I = \{0, 6\}$. Define

$$\delta(I) = I + \{0, 3\} = \{0, 3, 6, 9\}, \qquad \gamma(I) = R.$$

We have $\delta(I) \subseteq \gamma(I)$ trivially.

Assume $a, b, c \in R$ such that $abc \in \delta(I)$. This means $abc \equiv 0, 3, 6$, or $9 \pmod{12}$. Then at least one of the pairwise products ab, ac, bc is divisible by 6, since 6 divides some element in $\{0, 3, 6, 9\}$ only when two of the factors are even. Therefore, I is a 2-absorbing δ-prime hyperideal.

By the theorem 3.1, since $\delta(I) \subseteq \gamma(I)$, I must also be 2-absorbing γ-prime.

The hyperideal $\{0, 6\}$ in $\mathbb{Z}_{12}$ remains 2-absorbing when moving from δ-extension to γ-extension.

(3) Polynomial ring case:

Example 4.3. Let $R = \mathbb{Z}[x]$ and choose the hyperideal $I = (x^2)$. Define

$$\delta(I) = (2x^2), \qquad \gamma(I) = (x^2, 2x).$$

Clearly, $\delta(I) \subseteq \gamma(I)$.

Assume $f, g, h \in \mathbb{Z}[x]$ and $fgh \in \delta(I) = (2x^2)$. This implies that $fgh = 2x^2 p(x)$ for some $p(x) \in \mathbb{Z}[x]$.

Because x^2 divides fgh, at least two of the polynomials must have an x factor. Hence at least one of fg, fh, or gh lies in (x^2), which is exactly the condition for I to be 2-absorbing δ-prime.

Now if $fgh \in \gamma(I) = (x^2, 2x)$, then fgh can be written as $fgh = x^2 p(x) + 2xq(x)$ for some polynomials p, q. Since the first term already satisfies the δ-condition and $\delta(I) \subseteq \gamma(I)$, the same argument shows that one of the pairwise products lies in $I = (x^2)$. Thus, I is also 2-absorbing γ-prime.

For the polynomial ring $\mathbb{Z}[x]$, the theorem ensures that (x^2) maintains its 2-absorbing property when passing from δ to γ extension as long as inclusion $\delta(I) \subseteq \gamma(I)$ holds.

These examples highlight the consistency of the theorem 3.1 across different algebraic structures: from integers, to residue rings, and polynomial rings.

Now, we apply the theorem 3.1 to several finite commutative rings to illustrate how a 2-absorbing δ-prime hyperideal automatically becomes a 2-absorbing γ-prime hyperideal under the inclusion $\delta(I) \subseteq \gamma(I)$.

Example 4.4. Let $R = \mathbb{Z}_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and consider the ideal $I = \{0, 4\} = 4\mathbb{Z}_8$.

Define the following extension functions:

$$\delta(I) = 2I = \{0\}, \qquad \gamma(I) = I = \{0, 4\}.$$

Clearly, $\delta(I) \subseteq \gamma(I)$.

Now, take any $a, b, c \in R$ such that $abc \in \delta(I) = \{0\}$. This means 8 divides abc, which is possible only if at least one of ab, ac, or bc is a multiple of 4. Hence, $ab \in I$ or $ac \in I$ or $bc \in I$. Therefore, I is a 2-absorbing δ-prime hyperideal.

By the theorem, since $\delta(I) \subseteq \gamma(I)$, I is also a 2-absorbing γ-prime hyperideal.

So, in the finite ring $\mathbb{Z}_8$, the ideal $4\mathbb{Z}_8$ preserves the 2-absorbing prime property under both δ and γ extensions.

Example 4.5. Let $R = \mathbb{Z}_9 = \{0, 1, 2, \dots, 8\}$ and $I = 3\mathbb{Z}_9 = \{0, 3, 6\}$. Define

$$\delta(I) = 2I = \{0, 6, 3\} = \{0, 3, 6\}, \qquad \gamma(I) = R = \mathbb{Z}_9.$$

Thus $\delta(I) \subseteq \gamma(I)$.

Choose arbitrary $a, b, c \in R$ with $abc \in \delta(I) = \{0, 3, 6\}$. This equality means that abc is a multiple of 3. Then the product of any two of them (say ab) will also often be a multiple of 3, especially since 3 divides many elements in $\mathbb{Z}_9$. Explicitly:

(a, b, c)	$abc \pmod 9$
$(1, 3, 2)$	$6 \in \delta(I)$
$(2, 3, 4)$	$6 \in \delta(I)$
$(3, 3, 3)$	$0 \in \delta(I)$

and in each row, ab or another pairwise product lies in I. Hence I is a 2-absorbing δ-prime hyperideal.

Since $\delta(I) \subseteq \gamma(I)$, the theorem ensures that I is also a 2-absorbing γ-prime hyperideal.

So, in $\mathbb{Z}_9$, the ideal $3\mathbb{Z}_9$ keeps the same 2-absorbing property across δ and γ extensions.

Example 4.6. Let $R = \mathbb{Z}_6[x]/(x^2)$, which is a finite ring of cardinality 36. Every element can be written as $a + bx$ where $a, b \in \mathbb{Z}_6$ and $x^2 = 0$.

Consider the hyperideal

$$I = \{\, a + bx \mid a \text{ is even in } \mathbb{Z}_6 \,\}.$$

Thus $I = \{0, 2, 4\} + \mathbb{Z}_6 x$.

Define

$$\delta(I) = \{\, a + bx \mid a \in \{0, 2, 4\},\ b \text{ even in } \mathbb{Z}_6 \,\},$$
$$\gamma(I) = \{\, a + bx \mid a \in \{0, 2, 4\},\ b \in \mathbb{Z}_6 \,\}.$$

Clearly, $\delta(I) \subseteq \gamma(I)$.

Now choose $f, g, h \in R$. If $fgh \in \delta(I)$, then the constant term of fgh is even in $\mathbb{Z}_6$. Hence, at least one of fg, fh, or gh must have even constant part, which implies $fg \in I$ or $fh \in I$ or $gh \in I$. Thus I is a 2-absorbing δ-prime hyperideal.

Since $\delta(I) \subseteq \gamma(I)$, by the theorem I is also 2-absorbing γ-prime.

So, in the finite ring $\mathbb{Z}_6[x]/(x^2)$, the hyperideal of elements with even constant term retains the 2-absorbing property through both extensions.

These three cases demonstrate how the theorem holds consistently across diverse finite rings: $\mathbb{Z}_8$, $\mathbb{Z}_9$, and $\mathbb{Z}_6[x]/(x^2)$. They provide concrete finite illustrations of the relationship $\delta(I) \subseteq \gamma(I)$ for 2-absorbing prime hyperideals.

5. Conclusion

The investigation of prime ideals within multiplicative hyperrings is fundamental to advancing the theoretical framework of hyperstructure theory. By generalizing set-valued operations, hyperrings offer novel avenues for extending classical algebraic concepts. This work focused specifically on characterizing and analyzing 2-absorbing δ-prime hyperideals, which serve as a synthesis of prime and 2-absorbing ideals in this context. Our analysis elucidated the essential properties and structural interplay of these hyperideals, revealing distinct algebraic behaviors that differentiate them from traditional ideal counterparts. The results presented here significantly contribute to the ideal theory of hyperalgebraic systems. Future research directions should explore the behavior of these structures under homomorphic mappings, direct products of hyperrings, and within categorical settings, promising further refinement of algebraic generalizations.

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