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Note on "Mathematical modeling of dynamics behavior of terrorism and control"

Morad Ali Peyvand [1]
[1] Department of Mathematics, Yasouj University, Yasouj, Iran

Abstract. This note deals with some flaws in a recent paper by Gambo and Olarewaju (CJMS. 9(1), 2020, 68-85). It pinpoints the logic error in the proof of Theorem 3.2 in that paper and discusses some corrective works.

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1. Introduction

Recently, Gambo and Olarewaju [1] constructed a mathematical model of dynamics behavior of terrorism, represented by a system of differential equations, and used it to study the military/dialogue strategies to control the spread of terrorism. In that model, the total population at time t, denoted by $N(t)$, is separated into the six classes namely: Susceptible $S(t)$, Moderate $I(t)$, Terrorist $T(t)$, Leader $T_L(t)$, Foot soldiers $T_S(t)$, and Detention facilities $Q_T(t)$. So, $N(t) = S(t) + I(t) + T(t) + T_S(t)+T_L(t)+Q_T(t)$. The system of equations for the possible dynamics within the classes is as follows.

[1]Corresponding author: peyvand@yu.ac.ir
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$$\frac{dS}{dt} = \Lambda + aQ_T - eS - \mu S - \beta_1 \frac{T}{N} S$$
$$\frac{dI}{dt} = \beta_1 \frac{T}{N} S - \mu I - \beta_2 \frac{T}{N} I$$
$$\frac{dT}{dt} = \beta_2 \frac{T}{N} I + \delta Q_T + eS - bT - \pi\alpha T - (\mu + d)T - (1-k)\alpha T$$
$$\frac{dT_S}{dt} = \pi\alpha T - cT_S - (\mu + d)T_S$$
$$\frac{dT_L}{dt} = (1-k)\alpha T - (\mu + d)T_L$$
$$\frac{dQ_T}{dt} = bT + cT_S - (a + \delta + \mu + \eta)Q_T. \tag{1.1}$$

For the description of the parameters in the model, refer to [1], pp. 73-74.

It is well known that positively invariant sets play an important role in the analysis of some different aspects of dynamical systems, such as the existence, uniqueness and stability of solutions and so on. With respect to the proposed system (1.1), the set

$$\Omega = \{(S, I, T, T_S, T_L, Q_T) \in \mathbb{R}^6_+ \mid N \leq \frac{\Lambda}{\mu}\}$$

is provided, where

$$\mathbb{R}^6_+ = \{(S, I, T, T_S, T_L, Q_T) \in \mathbb{R}^6 \mid S, I, T, T_S, T_L, Q_T \geq 0\}.$$

Since

$$N = S + I + T + T_S + T_L + Q_T,$$

so,

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dT}{dt} + \frac{dT_S}{dt} + \frac{dT_L}{dt} + \frac{dQ_T}{dt}. \tag{1.2}$$

The authors [1] asserted that (1.2) can be reduced to

$$\frac{dN}{dt} = \Lambda - \mu N. \tag{1.3}$$

They constructed the proof of *Theorem 3.2* based on this assertion to show the boundedness and positively invariance of Ω (see [1] pp. 74-75).

Note that the equation (1.3)**, as a first order linear differential equation, is not the correct result of the equation** (1.2)**, so the proof of *Theorem 3.2* in** [1] **is wrong, and the use of *Lemma 3.1* is inappropriate in that work.**

The following section presents the correct result of the equation (1.2) and proposes a new theorem to show the boundedness and positively invariance of Ω.

2. POSITIVELY INVARIANT RIGION

To show that all the solution trajectories of the system (1.1) initiating inside Ω remain in the interior of Ω, we shall propose a new theorem that is a remedial work on some flaws in [1] discussed in the previous section.

Theorem 2.1. *The region Ω is positively invariant and attracts all solutions of the model.*

Proof. Let $\mathbb{R}^6_+$ be the nonnegative cone in the six-dimensional Euclidean space. From the system (1.1), we see that

$$\begin{aligned} \frac{dS}{dt}\,|_{S=0} &= \Lambda + aQ_T > 0, & \frac{dI}{dt}\,|_{I=0} &= \beta_1 \frac{T}{N} S > 0, \\ \frac{dT}{dt}\,|_{T=0} &= \delta Q_T + eS > 0, & \frac{dT_S}{dt}\,|_{T_S=0} &= \pi\alpha T > 0, \\ \frac{dT_L}{dt}\,|_{T_L=0} &= (1-k)\alpha T > 0, & \frac{dQ_T}{dt}\,|_{Q_T=0} &= bT + cT_S > 0 \end{aligned}$$

and S, I, T, T_S, T_L, Q_T are continuous functions of t. So, the vector field on each bounding hyperplane of $\mathbb{R}^6_+$ is pointing inward direction of $\mathbb{R}^6_+$. As such, all the solutions of the system (1.1) initiating in $\mathbb{R}^6_+$ will always remain there for all future time, i.e. $\mathbb{R}^6_+$ is positively invariant with respect to the system (1.1). In addition, it is easy to see that (1.2) can be reduced to

$$\frac{dN}{dt} = \Lambda - \mu(S + I + T + T_S + T_L + Q_T) - d(T + T_S + T_L) - \eta Q_T,$$

which implies that

$$\frac{dN}{dt} = \Lambda - \mu N - d(T + T_S + T_L) - \eta Q_T.$$

Then, $\frac{dN}{dt} < \Lambda - \mu N$. Applying Birkhoff's and Rota's theorems on this differential inequality [2, 3], as $t \to \infty$, we have $0 \le N(t) \le \frac{\Lambda}{\mu}$. Therefore, the solution of system (1.1) is bounded. Hence, any solution of the system originating from Ω remains in Ω. □

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