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On the stability of multi m-Jensen mappings

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Abstract. In this article, we unify the system of functional equations defining a multi m-Jensen mapping to a single equation. Using a fixed point theorem, we study the generalized Hyers-Ulam stability of such equation. As a result, we show that the multi m-Jensen mappings are hyperstable.

Keywords: Banach space, (multi) m-Jensen mapping, Hyers-Ulam stability.

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1. Introduction

Speaking of the stability of a functional equation we follow the question of Ulam: "when is it true that the solution of an equation differing slightly from a given one, must of necessity be close to the solution of the given equation?". The first stability problem concerning group homomorphisms was raised by Ulam [23] in 1940 and affirmatively solved by Hyers [12]. The result of Hyers was generalized by Rassias [20] for approximate linear mappings by allowing the Cauchy difference operator $f(x+y)-f(x)-f(y)$ to be controlled by $\epsilon(\|x\|^p+\|y\|^p)$. In 1994, a

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generalization of Rassias theorem was obtained by Găvruţa [11], who replaced $\epsilon(\|x\|^p+\|y\|^p)$ by a general control function $\varphi(x,y)$ by following Rassias' approach.

The stability of the Jensen functional equation

$$J\left(\frac{x+y}{2}\right)=\frac{J(x)+J(y)}{2} \tag{1.1}$$

was studied by a number of mathematicians (see for instance [13], [14], [15], [17] and [22]), whereas the stability of bi-Jensen equation was investigated by Bae and Park [1] and Jun et al., [16].

Prager and Schwaiger [19] introduced the notion of multi-Jensen mappings $f : V^n \longrightarrow W$ (V and W being vector spaces over the rationals) with the connection with generalized polynomials and obtained their general form. The aim of this note was to study the stability of the multi-Jensen equation. After that, the stability of multi-Jensen mappings in various normed spaces have been investigated by a number of authors (see [9], [10], [18] and [24]).

Let V be a commutative group, W be a linear space, and $n \geq 2$ be an integer. In this paper, we define the multi m-Jensen mappings $f : V^n \longrightarrow W$ which are m-Jensen in each variable, that is f satisfies the equation

$$J\left(\frac{z_1+\ldots+z_m}{m}\right)=\frac{J(z_1)+\ldots+J(z_m)}{m} \qquad (m>2). \tag{1.2}$$

in each variable and then present a characterization of such mappings. In other words, we reduce the system of n equations defining the multi m-Jensen mappings to obtain a single functional equation. We also prove the generalized Hyers-Ulam stability for multi m-Jensen functional equations by using the fixed point method which was used for the first time by Brzdęk in [6]; for more applications of this approach for the stability of various multi-mappings in Banach spaces see [2], [4], [5] and [21].

2. Characterization of multi m-Jensen mappings

Throughout this paper, $\mathbb{N}$ and $\mathbb{Q}$ stand for the set of all positive integers and rationals, respectively. In addition, $\mathbb{N}_0 := \mathbb{N}\cup\{0\}$ and $\mathbb{R}_+ := [0,\infty)$. For any $l \in \mathbb{N}_0, n \in \mathbb{N}$, $t=(t_1,\ldots,t_n)\in\{0,1\}^n$ and $x=(x_1,\ldots,x_n)\in V^n$ we write $lx := (lx_1,\ldots,lx_n)$ and $tx := (t_1x_1,\ldots,t_nx_n)$, where ra stands, as usual, for the rth power of an element a of the linear space V.

In what follows, let V and W be vector spaces over the rationals, $n\in\mathbb{N}$ and $x_j^n=(x_{1j},x_{2j},\ldots,x_{nj})\in V^n$, where $j\in\{1,\ldots,m\}$. We shall denote x_j^n by x_j or simply x if there is no risk of ambiguity. A mapping $f : V^n \longrightarrow W$ is called multi m-Jensen if f is m-Jensen in all variables.

In this section, we wish to show that the mapping $f : V^n \longrightarrow W$ is multi m-Jensen if and only if it satisfies the equation

$$m^n f\left(\frac{x_1+\ldots+x_m}{m}\right) = \sum_{l_1,\ldots,l_n\in\{1,\ldots,m\}} f(x_{1l_1}, x_{2l_2}, \ldots, x_{nl_n}) \tag{2.1}$$

for all $x_j = (x_{1j}, x_{2j}, \ldots, x_{nj}) \in V^k$ where $j \in \{1,\ldots,m\}$.

Here, we reduce the system of n equations defining the multi m-Jensen mapping to obtain a single functional equation.

Theorem 2.1. *Let $n \in \mathbb{N}$. Then, a mapping $f : V^n \longrightarrow W$ is multi m-Jensen mapping if and only if f satisfies the equation* (2.1)*.*

Proof. Suppose that f is a multi m-Jensen mapping. We prove f satisfies the equation (2.1) by induction on n. For $n = 1$, it is trivial that f satisfies the equation (1.2). Assume that (2.1) is valid for some positive integer $n > 1$. Then,

$$\begin{aligned}
& f\left(\frac{x_1^{n+1}+\ldots+x_m^{n+1}}{m}\right)\\
&= f\left(\frac{x_{11}+\ldots+x_{1m}}{m}, \ldots, \frac{x_{n1}+\ldots+x_{nm}}{m}, \frac{x_{n+11}+\ldots+x_{n+1m}}{m}\right)\\
&= \frac{1}{m^n} \sum_{l_1,\ldots,l_n\in\{1,\ldots,m\}} f\left(x_{1l_1}, x_{2l_2}, \ldots, x_{nl_n}, \frac{x_{n+11}+\ldots+x_{n+1m}}{m}\right)\\
&= \frac{1}{m^{n+1}} \sum_{l_1,\ldots,l_{n+1}\in\{1,\ldots,m\}} f\left(x_{1l_1}, x_{2l_2}, \ldots, x_{n+1l_{n+1}}\right).
\end{aligned}$$

This means that (2.1) holds for $n+1$.

Conversely, assume that f satisfies the equation (2.1). Now, fix $j \in \{1,\ldots,n\}$, put $z_k = x_{kl_k}$ and for all $k \in \{1,\ldots,n\}\backslash\{j\}$, where $l_k \in \{1,\ldots,m\}$. We have

$$\begin{aligned}
& f\left(z_1,\ldots,z_{j-1}, \frac{x_{j1}+\ldots+x_{jm}}{m}, z_{j+1},\ldots,z_n\right)\\
&= \frac{1}{m^n} m^{n-1} \sum_{l_j\in\{1,\ldots,m\}} f\left(z_1,\ldots,z_{j-1}, x_{jl_j}, z_{j+1},\ldots,z_n\right)\\
&= \frac{1}{m} \sum_{l_j\in\{1,\ldots,m\}} f\left(z_1,\ldots,z_{j-1}, x_{jl_j}, z_{j+1},\ldots,z_n\right).
\end{aligned}$$

It follows from the above relation that f is m-Jensen in the jth variable. Since j is arbitrary, we obtain the desired result. □

Recall that a mapping $f : V^n \longrightarrow W$ is called *multi-additive* if it is additive (satisfies the Cauchy's functional equation $A(x+y) = A(x) + A(y)$) in each variable.

Proposition 2.2. *Let $n \in \mathbb{N}$. Consider the mapping $f : V^n \longrightarrow W$. If there exists a multi-additive mapping $g : V^n \longrightarrow W$ such that $f(x) = g(x) + f(0)$ in which $f(0)$ is a constant, then f is multi m-Jensen.*

Proof. Fix $j \in \{1, \ldots, n\}$. We have

$$\begin{aligned}
& f\left(z_1, \ldots, z_{j-1}, \frac{x_{j1} + \ldots + x_{jm}}{m}, z_{j+1}, \ldots, z_n\right) \\
& = g\left(z_1, \ldots, z_{j-1}, \frac{x_{j1} + \ldots + x_{jm}}{m}, z_{j+1}, \ldots, z_n\right) + f(0, \ldots, 0) \\
& = \frac{1}{m} \sum_{l_j=1}^{m} g\left(z_1, \ldots, z_{j-1}, x_{jl_j}, z_{j+1}, \ldots, z_n\right) + f(0, \ldots, 0) \\
& = \frac{1}{m} \sum_{l_j=1}^{m} \left[g\left(z_1, \ldots, z_{j-1}, x_{jl_j}, z_{j+1}, \ldots, z_n\right) + f(0, \ldots, 0)\right] \\
& = \frac{1}{m} \sum_{l_j=1}^{m} f\left(z_1, \ldots, z_{j-1}, x_{jl_j}, z_{j+1}, \ldots, z_n\right). \qquad (2.2)
\end{aligned}$$

Thus, g is m-Jensen in the jth variable. This completes the proof. □

3. Stability of multi m-Jensen mappings

In this section, we prove the generalized Hyers-Ulam stability of equation (2.1) by a fixed point result (Theorem 3.1) in Banach spaces. Throughout, for two sets X and Y, the set of all mappings from X to Y is denoted by Y^X. Here, we introduce the oncoming three hypotheses:

(A1) Y is a Banach space, $\mathcal{S}$ is a nonempty set, $j \in \mathbb{N}$, $g_1, \ldots, g_j : \mathcal{S} \longrightarrow \mathcal{S}$ and $L_1, \ldots, L_j : \mathcal{S} \longrightarrow \mathbb{R}_+$,

(A2) $\mathcal{T} : Y^{\mathcal{S}} \longrightarrow Y^{\mathcal{S}}$ is an operator satisfying the inequality

$$\|\mathcal{T}\lambda(x) - \mathcal{T}\mu(x)\| \leq \sum_{i=1}^{j} L_i(x) \|\lambda(g_i(x)) - \mu(g_i(x))\|, \quad \lambda, \mu \in Y^{\mathcal{S}}, x \in \mathcal{S},$$

(A3) $\Lambda : \mathbb{R}_+^{\mathcal{S}} \longrightarrow \mathbb{R}_+^{\mathcal{S}}$ is an operator defined through

$$\Lambda\delta(x) := \sum_{i=1}^{j} L_i(x)\delta(g_i(x)) \qquad \delta \in \mathbb{R}_+^{\mathcal{S}}, x \in \mathcal{S}.$$

In the following, we present a result in fixed point theory [7, Theorem 1] which plays a key tool to obtain our aim in this paper.

Theorem 3.1. *Let hypotheses* (A1)-(A3) *hold and the function* $\theta : \mathcal{S} \longrightarrow \mathbb{R}_+$ *and the mapping* $\phi : \mathcal{S} \longrightarrow Y$ *fulfills the following two conditions:*

$$\|\mathcal{T}\phi(x) - \phi(x)\| \leq \theta(x), \quad \theta^*(x) := \sum_{l=0}^{\infty} \Lambda^l \theta(x) < \infty \qquad (x \in \mathcal{S}).$$

Then, there exists a unique fixed point ψ *of* $\mathcal{T}$ *such that*

$$\|\phi(x) - \psi(x)\| \leq \theta^*(x) \qquad (x \in \mathcal{S}).$$

Moreover, $\psi(x) = \lim_{l\to\infty} \mathcal{T}^l \phi(x)$ *for all* $x \in \mathcal{S}$.

Here and subsequently, for the mapping $f : V^n \longrightarrow W$, we consider the difference operator $\mathcal{J}f : V^n \times V^n \longrightarrow W$ by

$$\mathcal{J}f(x_1, \ldots, x_m) := m^n f\left(\frac{x_1 + \ldots + x_m}{m}\right) - \sum_{l_1,\ldots,l_n \in \{1,\ldots,m\}} f(x_{1l_1}, x_{2l_2}, \ldots, x_{nl_n})$$

for all $x_j = (x_{1j}, x_{2j}, \ldots, x_{nj}) \in V^k$ where $j \in \{1, \ldots, m\}$.

We recall the upcoming lemma from [3] which will be useful in the proof of our stability result. For simplicity, given a $k \in \mathbb{N}$, we write $S := \{0,1\}^k$, and S_i stands for the set of all elements of S having exactly i zeros, i.e.,

$$S_i : \{(s_1, \ldots, s_k) \in S : \mathrm{card}\{j : \ s_j = 0\} = i\}, \qquad i \in \{0, \ldots, k\}.$$

Lemma 3.2. *Let* $k \in \mathbb{N}$, $l \in \mathbb{N}_0$ *and* $\psi : S \longrightarrow \mathbb{R}$. *Then*

$$\sum_{v=0}^{k} \sum_{w=0}^{k} \sum_{s \in S_w} \sum_{t \in S_v} (2^l - 1)^w \psi(st) = \sum_{i=0}^{k} \sum_{p \in S_i} (2^{l+1} - 1)^i \psi(p).$$

From now on, S stands for $\{0,1\}^n$ and $S_i \subseteq S$ for $i \in \{0, \ldots, n\}$. We have the following stability result for the functional equation (2.1).

Theorem 3.3. *Let* V *be a linear space and* W *be a Banach space. Suppose that* $\phi : \overbrace{V^n \times \cdots \times V^n}^{m-\text{times}} \longrightarrow \mathbb{R}_+$ *is a function satisfying the equality*

$$\lim_{l\to\infty} \left(\frac{1}{m^n}\right)^l \sum_{i=0}^{n} \sum_{p \in S_i} (2^l - 1)^i \phi(m^l p x_1, \ldots, m^l p x_m) = 0 \tag{3.1}$$

for all $x_1, \ldots, x_m \in V^n$. *Assume also* $f : V^n \longrightarrow W$ *is a mapping satisfying the inequality*

$$\|\mathcal{J}f(x_1, \ldots, x_m)\| \leqslant \phi(x_1, \ldots, x_m) \tag{3.2}$$

for all $x_1, \ldots, x_m \in V^n$. *If*

$$\Phi(x) =: \sum_{l=0}^{n} \left(\frac{1}{m^n}\right)^{l+1} \sum_{i=0}^{n} \sum_{p \in S_i} (2^l - 1)^i \phi(m^l px, 0, \ldots, 0) < \infty, \quad (3.3)$$

for all $x \in V^n$, *then there exists a unique solution* $\mathcal{F} : V^n \longrightarrow W$ *of* (2.1) *such that*

$$\|f(x) - \mathcal{F}(x)\| \leq \Phi(x) \quad (3.4)$$

for all $x \in V^n$.

Proof. Replacing $x = x_1$ by mx and putting $x_k = (0, \ldots, 0)$ for $k \in \{2, \ldots, m\}$ in (3.2), we have

$$\left\| f(x) - \frac{1}{m^n} \sum_{s \in S} f(smx) \right\| \leq \frac{1}{m^n} \phi(x, 0, \ldots, 0) \quad (3.5)$$

where $x \in V^n$. Set $\theta(x) := \frac{1}{m^n}\phi(x, 0, \ldots, 0)$ and $\mathcal{T}\theta(x) := \frac{1}{m^n} \sum_{s \in S} f(smx)$ where $\theta \in W^{V^n}, x \in V^n$. Then, the relation (3.5) can be modified as

$$\|f(x) - \mathcal{T} f(x)\| \leq \theta(x) \qquad (x \in V^n). \quad (3.6)$$

Define $\Lambda\eta(x) := \frac{1}{m^n} \sum_{s \in S} \eta(smx)$ for all $\eta \in \mathbb{R}_+^{V^n}, x \in V^n$. We now see that Λ has the form described in (A3) with $\mathcal{S} = V^n$, $g_i(x) = g_s(x) = smx$ and $L_i(x) = \frac{1}{m^n}$ for all i and $x \in V^n$. Furthermore, for each $\lambda, \mu \in W^{V^n}$ and $x \in V^n$, we get

$$\begin{aligned} \|\mathcal{T}\lambda(x) - \mathcal{T}\mu(x)\| &= \left\| \frac{1}{m^n} \left[\sum_{s \in S} (\lambda(smx) - \mu(smx)) \right] \right\| \\ &\leq \frac{1}{m^n} \sum_{s \in S} \|\lambda(smx) - \mu(smx)\| . \end{aligned}$$

The above relation shows that the hypothesis (A2) holds. By induction on l, one can check for any $l \in \mathbb{N}_0$ and $x \in V^n$ that

$$\Lambda^l \theta(x) := \left(\frac{1}{m^n}\right)^l \sum_{i=0}^{n} (2^l - 1)^i \sum_{p \in S_i} \theta(m^l px). \quad (3.7)$$

Fix an $x \in V^n$. Here, we adopt the convention that $0^0 = 1$. Hence, the relation (3.7) is trivially true for $l = 0$. Next, assume that (3.7) holds for a $l \in \mathbb{N}_0$. Then, using Lemma 3.2 for $k = n$ and $\psi(s) := \theta(m^{l+1} sx)$

for $s \in S$, we find

$$\begin{aligned}\Lambda^{l+1}\theta(x) = \Lambda(\Lambda^l\theta)(x) &= \frac{1}{m^n}\sum_{v=0}^{n}\sum_{t\in S_v}(\Lambda^l\theta)(mtx)\\ &= \left(\frac{1}{m^n}\right)^{l+1}\sum_{v=0}^{n}\sum_{t\in S_v}\sum_{w=0}^{n}(2^l-1)^w\sum_{s\in S_w}\theta(m^{l+1}stx)\\ &= \left(\frac{1}{m^n}\right)^{l+1}\sum_{v=0}^{n}\sum_{w=0}^{n}\sum_{s\in S_w}\sum_{t\in S_v}(2^l-1)^w\theta(m^{l+1}stx)\\ &= \left(\frac{1}{m^n}\right)^{l+1}\sum_{i=0}^{n}\sum_{p\in S_i}(2^{l+1}-1)^i\theta(m^{l+1}px).\end{aligned}$$

Therefore, (3.7) holds for any $l \in \mathbb{N}_0$ and $x \in V^n$. The relations (3.3) and (3.7) necessitate that all assumptions of Theorem 3.1 are satisfied. Hence, there exists a mapping $\mathcal{F} : V^n \longrightarrow W$ such that

$$\mathcal{F}(x) = \lim_{l\to\infty}(\mathcal{T}^l f)(x) = \frac{1}{m^n}\sum_{s\in S}\mathcal{F}(smx) \qquad (x \in V^n),$$

and also (3.4) holds. We shall to show that

$$\|\mathcal{D}(\mathcal{T}^l f)(x_1,\ldots,x_m)\| \le \left(\frac{1}{m^n}\right)^l\sum_{i=0}^{n}\sum_{p\in S_i}(2^l-1)^i\phi(m^lpx_1,\ldots,m^lpx_m) \tag{3.8}$$

for all $x_1,\ldots,x_m \in V^n$ and $l \in \mathbb{N}_0$. We argue by induction on l. The inequality (3.8) is valid for $l = 0$ by (3.2). Assume that (3.8) is true for an $l \in \mathbb{N}_0$. Then

$$\begin{aligned}&\|\mathcal{D}(\mathcal{T}^{l+1} f)(x_1,\ldots,x_m)\|\\ &= \frac{1}{m^n}\left\|\sum_{s\in S}\mathcal{D}(\mathcal{T}^l f)(smx_1,\ldots,smx_m)\right\|\\ &\le \left(\frac{1}{m^n}\right)^{l+1}\sum_{s\in S}\sum_{i=0}^{n}\sum_{t\in S_i}(2^l-1)^i\phi(m^{l+1}stx_1,\ldots,m^{l+1}stx_m)\\ &= \left(\frac{1}{m^n}\right)^{l+1}\sum_{i=0}^{n}\sum_{p\in S_i}(2^{l+1}-1)^i\phi(m^{l+1}px_1,\ldots,m^{l+1}px_m)\end{aligned}$$

for all $x_1,\ldots,x_m \in V^n$. We note that the last equality follows from Lemma 3.2 with $k := n$ and $\psi(s) := \phi(m^{l+1}sx_1,\ldots,m^{l+1}sx_m)$ $(s \in S)$. Letting $l \to \infty$ in (3.8) and applying (3.1), we arrive at $\mathcal{D}\mathcal{F}(x_1,\ldots,x_m) =$

0 for all $x_1, \ldots, x_m \in V^n$. This means that the mapping $\mathcal{F}$ satisfies (2.1). Lastly, assume that $\mathfrak{F} : V^n \longrightarrow W$ is another mapping satisfying the equation (2.1) and inequality (3.4), and fix $x \in V^n$, $j \in \mathbb{N}$. By the relation (3.3), we have

$$\begin{aligned}
&\|\mathcal{F}(x) - \mathfrak{F}(x)\| \\
&= \left\| \left(\frac{1}{m^n}\right)^j \mathcal{F}(2^j x) - \left(\frac{1}{m^n}\right)^j \mathfrak{F}(2^j x) \right\| \\
&\leq \left(\frac{1}{m^n}\right)^j (\|\mathcal{F}(2^j x) - f(2^j x)\| + \|\mathfrak{F}(2^j x) - f(2^j x)\|) \\
&\leq 2 \left(\frac{1}{m^n}\right)^j \Phi(2^j x) \\
&\leq 2 \left(\frac{1}{m^n}\right)^j \sum_{l=j}^{\infty} \left(\frac{1}{m^n}\right)^{l+1} \sum_{i=0}^{n} \sum_{p \in S_i} (2^l - 1)^i \phi(m^l p x, 0, \ldots, 0).
\end{aligned}$$

Consequently, letting $j \to \infty$ and using the fact that series (3.3) is convergent for all $x \in V^n$, we obtain $\mathcal{F}(x) = \mathfrak{F}(x)$ for all $x \in V^n$, which finishes the proof. □

In the sequel, $\begin{pmatrix} n \\ k \end{pmatrix}$ is the binomial coefficient defined for all $n, k \in \mathbb{N}_0$ with $n \geq k$ by $n!/(k!(n-k)!)$. In the next corollary, we show that the functional equation (2.1) is stable.

Corollary 3.4. *Let $\delta > 0$. Let also V be a normed space and W be a Banach space. If $f : V^n \longrightarrow W$ is a mapping satisfying the inequality*

$$\|\mathcal{J} f(x_1, \ldots, x_m)\| \leq \delta$$

for all $x_1, \ldots, x_m \in V^n$, then there exists a unique solution $\mathcal{F} : V^n \longrightarrow W$ of (2.1) such that

$$\|f(x) - \mathcal{F}(x)\| \leq \frac{\delta}{m^n - 2^n}$$

for all $x \in V^n$.

Proof. Setting the constant function $\phi(x_1,\ldots,x_m)=\delta$ for all $x_1,\ldots,x_m\in V^n$, and applying Theorem 3.3, we have

$$\begin{aligned}\Phi(x)&=\sum_{l=0}^{\infty}\left(\frac{1}{m^n}\right)^{l+1}\sum_{i=0}^{n}\sum_{p\in S_i}(2^l-1)^i\phi(m^lpx,0\ldots,0)\\&=\delta\sum_{l=0}^{\infty}\left(\frac{1}{m^n}\right)^{l+1}\sum_{i=0}^{n}\left(\begin{array}{c}n\\i\end{array}\right)(2^l-1)^i\times 1^{n-i}\\&=\delta\sum_{l=0}^{\infty}\left(\frac{1}{m^n}\right)^{l+1}2^{nl}\\&=\frac{\delta}{m^n}\sum_{l=0}^{\infty}\left(\frac{2^n}{m^n}\right)^{l}\\&=\frac{\delta}{m^n-2^n}.\end{aligned}$$

□

We note that the above corollary is valid only for any integer m with $m>2$.

Let A be a nonempty set, (X,d) a metric space, $\psi\in\mathbb{R}_+^{A^n}$, and $\mathcal{F}_1,\mathcal{F}_2$ operators mapping a nonempty set $D\subset X^A$ into X^{A^n}. We say that operator equation

$$\mathcal{F}_1\varphi(a_1,\ldots,a_n)=\mathcal{F}_2\varphi(a_1,\ldots,a_n) \tag{3.9}$$

is ψ-hyperstable provided every $\varphi_0\in D$ satisfying inequality

$$d(\mathcal{F}_1\varphi_0(a_1,\ldots,a_n),\mathcal{F}_2\varphi_0(a_1,\ldots,a_n))\le\psi(a_1,\ldots,a_n),$$

fulfils (3.9) for all $a_1,\ldots,a_n\in A$. This definition is introduced in [8]. In other words, a functional equation $\mathcal{F}$ is *hyperstable* if any mapping f satisfying the equation $\mathcal{F}$ approximately is a true solution of $\mathcal{F}$. Under some conditions the functional equation (2.1) can be hyperstable as follows.

Corollary 3.5. *Suppose that $\delta_{ij}>0$ for $i\in\{1,\ldots,n\}$ and $j\in\{1,\ldots,m\}$ fulfill $\sum_{i=1}^n\sum_{j=1}^m\delta_{ij}<n$. Let V be a normed space and W be a Banach space. If $f:V^n\longrightarrow W$ is a mapping satisfying the inequality*

$$\|\mathcal{J}f(x_1,\ldots,x_m)\|\le\prod_{i=1}^{n}\prod_{j=1}^{m}\|x_{ij}\|^{\delta_{ij}}$$

for all $x_1,\ldots,x_m\in V^n$, then f is a multi m-Jensen mapping.

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