Caspian Journal of Mathematical Sciences (CJMS)
University of Mazandaran, Iran
http://cjms.journals.umz.ac.ir
ISSN: 1735-0611

CJMS. **1**(2)(2012), 104-108

Comments on Multiparameter Estimation in Truncated Power Series Distributions under the Stein's Loss

Riyadh R. Al-Mosawi [1]
Department of Mathematics, Thiqar University, Iraq

Abstract. This comment is to show that Theorem 3.3 of Dey and Chung (1991) (Multiparameter estimation in truncated power series distributions under the Stein's loss. *Commun. Statist.-Theory Meth.,* **20**, 309-326) may give us misleading results. Analytically and through simulation, we show that the Theorem does not improve the given estimator.

Keywords: Left-Truncated power series distributions, Stein loss function.

1. Introduction

Let $X = (X_1, \cdots, X_p)$ where $X_1, \cdots, X_p$ are p independent random variables, X_i having the following left-truncated power series distribution

$$P_{\theta_i}(x_i) = \begin{cases} g_i(\theta_i)t_i(x_i)\theta_i^{x_i}, & x_i = a_i, a_i+1, \cdots; \\ 0, & \text{otherwise,} \end{cases}$$

where a_i is nonzero positive integer and $g_i(\theta_i)$ is a normalizing constant, given as

$$g_i^{-1}(\theta_i) = \sum_{x_i=a_i}^{\infty} t_i(x_i)\theta_i^{x_i}, \ \theta_i > 0, i = 1, \cdots, p.$$

[1] Received: 15 May 2012
Revised: 12 June 2012
Accepted: 20 June 2012

Consider the loss function (Stein loss) is given by

$$L(\theta, \delta) = \sum_{i=1}^{p} \left(\frac{\delta_i}{\theta_i} - log\left(\frac{\delta_i}{\theta_i}\right) - 1 \right) \tag{1.1}$$

where $\delta = (\delta_1, \cdots, \delta_p)$ is an estimate of $\theta = (\theta_1, \cdots, \theta_p)$ and *log* denotes the natural logarithm. For the loss function (1.1), the best multiple estimator of θ (which is also the best unbiased estimator) is given by $\delta^0(X) = (\delta_1^0(X), \cdots, \delta_p^0(X))$ where

$$\delta_i^0(X) = \begin{cases} \dfrac{t_i(X_i - 1)}{t_i(X_i)}, & X_i = a_i + 1, a_i + 1, \cdots; \\ 0, & \text{elsewhere.} \end{cases}$$

and $t_i(a_i - 1)$ is defined zero.
Suppose the rival estimator of θ as

$$\begin{aligned} \delta(X) =& \delta(X) + \phi(X) \\ =& (\delta_1^0(X) + \phi_1(X), \cdots, \delta_p^0(X) + \phi_p(X)) \end{aligned}$$

where $\phi(X) = (\phi_1(X), \cdots, \phi_p(X)), \phi_i(X) > 0$ and $\phi_i(X) = 0$ if $X_i < a_i + 1, i = 1, \cdots, p$. Assume $\delta_i^0(X), i = 1, \cdots, p$, be an increasing function of X.
The following theorem and corollary are from Dey and Chung (1991).

Theorem 1.1. *Suppose that* $\delta(X) = \delta^0(X)(1 + \psi(X))$ *where* $\psi(X) = (\psi_1(X), \cdots, \psi_p(X))$ *and* $\psi_i(X) = \phi_i(X)/\delta_i^0(X), i = 1, \cdots, p$ *with*

$$\psi_i(X) = \frac{d(X)e^{-X_i}}{b + s_2}, \; s_2 = \sum_{j=1}^{p} e^{-2X_j}, i = 1, \cdots, p$$

and the following additional conditions hold

(1) $b \geq 1/4$
(2) $0 < d(X) < 1/2$
(3) $d(X)$ is a decreasing function in each coordinate
(4) $d(X + e_i) \leq e^{-2} d(X), i = 1, \cdots, p.$

Then $\delta(X)$ will dominate $\delta^0(X)$ in terms of risk if $p \geq 2$.

Corollary 1.2. *Suppose that* $\delta(X) = \delta^0(X)(1 + \psi(X))$ *where* $\psi(X) = (\psi_1(X), \cdots, \psi_p(X))$ *with*

$$\psi_i(X) = \frac{0.5e^{-2s}e^{-X_i}}{b + s_2}$$

where $s = \sum_{j=1}^{p} X_j$, $s_2 = \sum_{j=1}^{p} e^{-2X_j}$ and $b \geq 1/4$. Then for $p \geq 2$, $\delta(X)$ dominates $\delta^0(X)$ in terms of risk.
Now borrowing an idea of Liang (2009), we show that δ given in corollary

3.3.1 in fact is not better than δ^0 in terms of risk. For simplicity, consider $a_i = 1$ for all $i = 1, \cdots, p$ and let $\alpha = \min(\delta_1^0(2), \cdots, \delta_p^0(2))$. Suppose that the parameter space is given by $\Omega = \{\theta; \theta_i > 0, i = 1, \cdots, p\}$ and define the subspace $\Omega_0 \subset \Omega$ such that $\Omega_0 = \{\theta; \theta_i < \alpha, i = 1, \cdots, p\}$. The risk difference of $\delta(X)$ and $\delta^0(X)$ is given by

$$\begin{aligned}\Delta(\theta) =& R(\theta, \delta) - R(\theta, \delta^0) \\ =& R(\theta, \delta^0 + \phi) - R(\theta, \delta^0) \\ =& \sum_{i=1}^{p} E\left(\frac{\phi_i(X)}{\theta_i} - log\left(1 + \frac{\phi_i(X)}{\delta_i^0(X)}\right)\right).\end{aligned}$$

Since $\phi_i > 0$ and $\theta_i < \alpha$ for $i = 1, \cdots, p$ and $\theta \in \Omega_0$ and also for all x such that $x_i \geq 2, i = 1, \cdots, p$ we have

$$\begin{aligned}\delta_i^0(X) \geq& \min(\delta_1^0(X), \cdots, \delta_p^0(X)) \\ \geq& \min(\delta_1^0(2), \cdots, \delta_p^0(2)) \\ =& \alpha,\end{aligned}$$

then we get

$$\begin{aligned}\Delta(\theta) \geq& \sum_{i=1}^{p} E\left(\frac{\phi_i(X)}{\alpha} - log\left(1 + \frac{\phi_i(X)}{\alpha}\right)\right) \\ =& \sum_{i=1}^{p} E\Big(\eta_i(X) - log(1 + \eta_i(X))\Big),\end{aligned}$$

where $\eta_i(X) = \phi_i(X)/\alpha$. It is known that $\eta_i(X) - log(1 + \eta_i(X)) > 0$ for X such that $X_i \geq 2$ so that $\Delta(\theta) > 0$ for $\theta \in \Omega_0$.
Another way to show that the estimator δ is not better than δ^0 is by simulation. A Monti Carlo simulation is carried out to generate random variables from zero-truncated Poisson distribution using Matlab 7.4. For a particular set of parameters, the risks of the estimators δ^0 and δ are computed and reported in Table 1. From Table 1, we observe that the risk of δ is slightly higher than the risk of δ^0 for the a specific set of parameters and hence δ is not an improved estimator of δ^0.

Table 1. R_1 is the risk of δ^0 and R_2 is the risk of δ.

p	Parameters										R_1	R_2
2	.01	.07									5.5768	5.5782
3	.01	.07	.003								5.6621	5.6634
4	.01	.07	.003	.05							5.4648	5.4662
5	.01	.07	.003	.05	.00001						5.6188	5.6202
10	.01	.07	.003	.05	.00001	.001	.000015	.002	.001	.04	5.0567	5.0580

2. Main results

The following is an example of a definition.

Definition 2.1. Let X be a real Banach space. A non-empty closed set $P \subset X$ is called a cone of X if it satisfies the following conditions:
(1) $x \in P, \mu \geq 0$ implies $\mu x \in P$,
(2) $x \in P, -x \in P$ implies $x = 0$.

Here is an example of a table.

Table 2.

1	2	3
$f(x)$	$g(x)$	$h(x)$
a	b	c

The following is an example of an example.

Example 2.2. Consider the following boundary value problem system:

$$\begin{cases} u^{(4)}(t) = f(t, u(t), u''(t)) \qquad 0 \leq t \leq 1, \\ u(0) = u(1) = 0, \qquad u''(0) - u'''(0) = 0, \qquad u''(1) - \frac{1}{2}u''(\frac{1}{2}) = 0, \end{cases} \tag{2.1}$$

where $f(t, u(t), u''(t)) = \frac{1}{\sqrt{1+u}} - (u'')^{-3} + \sin \pi t$. Clearly,

$$0 < \int_0^1 (s + \frac{1}{2})(1 - s)ds < +\infty, \qquad \min f_0 = +\infty, \qquad \max f_\infty = 0.$$

System (2.1) has at least one positive solution.

The following is an example of a theorem and a proof [**?**, **?**].

Theorem 2.3. *If* $\mathbf{B}$ *is an open ball of a real inner product space* $\mathcal{X}$ *of dimension greater than ...*

Proof. First note that if f is a generalized Jensen mapping with parameters $t = s \geq r$, then

$$\begin{aligned} f(\lambda(x + y)) &= \lambda f(x) + \lambda f(y) \\ &\leq \lambda(f(x) + f(y)) \\ &= f(x) + f(y) \end{aligned} \tag{2.2}$$

for some $\lambda \geq 1$ ... in the proof of Theorem 2.3, one can show that $f(x) = f(y_0)$ ... □

The following is an example of a remark.

Remark 2.4. One can easily conclude that g is continuous by using Theorem 2.3.

Acknowledgments

The author is very grateful to the referee for his/her valuable suggestions and comments.

References

[1] Dey, D. K. and Chung, Y. (1991). Multiparameter estimation in truncated power series distributions under the Stein's loss. *Commun. Statist.-Theory Meth.,* **20**, 309-326.

[2] Liang, T. (2009). Comments on "Estimating the parameter of the population selected from discrete exponential family". *Statistics and Probability Letters,* **79,** 2208-2211.