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On Relation Between Two-criteria User-optimized Route Choice Problem and Vector Variational Inequality Problem in fuzzy Environment

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Abstract. A two-criteria user-optimized route choice problem is proposed, in which each user of a network system seeks to determine his/her optimal route of travel between an origin-destination (O-D) pair considering two-criteria simultaneously. In this problem, the two-criteria of travel, time and cost, between an O-D pair are fuzzy, in the sense that, time and cost of which links are chosen for traveling are uncertain. Applying the concept of α-cut level, a fuzzy vector disutility function on a path is computed. Furthermore, the fuzzy vector equilibrium principle as a generalization and extension of the Wardrop equilibrium principle is defined. Finally, by reducing this fuzzy principle to a crisp one, the relationship between the vector equilibrium flow and the solution of a vector variational inequality problem is discussed.

Keywords: Fuzzy equilibrium principle, Equilibrium flow, Vector variational inequality.

2000 Mathematics subject classification: 90B20; 90B50.

1. Introduction

In a user-optimized route choice problem (UORCP) each traveler seeks to optimize own individual preference. In the study of this problem, fundamental is the concept of equilibrium introduced by Wardrop [8]. It states that, in a network of different routes, the travel times on all

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routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route. Traditionally, the UORCP is formulated as numerous mathematical models such as variational inequality models; see for instance [1] and the references therein. In these works, UORCP is formulated as a single criteria model in which each user of a traffic network seeks to determine his cost-minimizing routes of travel between an O-D pair. Later, a two-criteria network equilibrium problem proposed in which travelers seek to select their optimal routes of travel considering two-criteria simultaneously, for example, travel cost and travel time[4, 6]. Recently, some researchers based on this assumption that travelers choose their travel route based on perceived travel time rather than the actual travel time, proposed the fuzzy user-optimized route choice problem(FUORCP)[2, 3] and the variational inequality approach adopted to formulate this problem; see for instance [7, 9]. This is where our interest in studying and modeling FUORCP originates. Motivated by these works, we define the fuzzy vector equilibrium principle, which is the generalization of the Wardrop equilibrium principle and the extension of this crisp equilibrium condition into a fuzzy environment. A major difficulty we encounter in studying fuzzy two-criteria equilibrium problems is that fuzzy arithmetical operations are rather computational-intensive and the operational definitions of the fuzzified version of the order relations are hard to come up with for practical UORCP.

In this paper, we shall be concerned with the most likely estimate of each link travel time and travel cost that are associated with the fuzzy link travel time and fuzzy link travel cost at an $\alpha - cut$ level to avoid the intensive computations. We shall construct a vector variational inequality problem (VVIP) and discuss the relation between the solution of this problem and the equilibrium flow of the UORCP.

The paper is structured as follows. In section 2, we briefly introduce some basic concepts of fuzzy set. In section 3, we propose a fuzzy two-criteria user-optimized route choice model. Moreover, applying the concept of α-cut, we define the fuzzy vector equilibrium principle. In section 4, we show at a certain α-cut level that taking a plausible point estimate of each route travel time and travel cost in the two-criteria user-optimized route choice problem yields a crisp user-optimized route choice problem which may have relations with vector variational inequality. Section 5 concludes this paper by making some remarks.

2. A Brief review of fuzzy set

A crisp set classifies all elements over the universe of discourse U into two groups: elements that certainly belong to the set and elements that certainly do not. Let A be a crisp set in U. The classification of members can be done using a indicator function $\mu_A : U \to \{0, 1\}$ that for an element x of U, $\mu_A(x) = 1$ if x belongs to A and $\mu_A(x) = 0$ if x does not. However, for a fuzzy set, a membership function $\mu_{\tilde{A}}$ of a which fuzzy set $\tilde{A}$ is a generalization of the characteristic function as, $\mu_{\tilde{A}} : U \to [0, 1]$. For an element x of U, the value $\mu_{\tilde{A}}(x)$ is called the membership degree of x in the fuzzy set $\tilde{A}$, which quantifies the grade of membership of x in $\tilde{A}$. Thus, the near the value of $\mu_{\tilde{A}}(x)$ is unity, the higher the grade of membership of x in $\tilde{A}$.

A fuzzy set $\tilde{A}$ can be characterized as a set of ordered pairs of element x and grade $\mu_{\tilde{A}}(x)$ and is often written $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in U\}$.

A fuzzy number $\tilde{M}$ is a convex normalized fuzzy set of the real line R^1 whose membership function is piecewise continuous. So, fuzzy number is expressed as a fuzzy set defining a fuzzy interval in the real number. Since the boundary of this interval is ambiguous, the interval is also a fuzzy set. The α-level set $\tilde{M}_\alpha$ of a fuzzy number $\tilde{M}$ can be represented by the closed interval which depends on the value of $\alpha \in [0, 1]$. Namely,

$$\tilde{M}_\alpha = \{x \in R^1 | \mu_{\tilde{M}}(x) \geq \alpha\} = [m_\alpha^L, m_\alpha^R]$$

where m_α^L and m_α^R are the lower bound and upper bound of the α-level set $\tilde{M}_\alpha$, respectively. As an example, a fuzzy number $\tilde{M}$ is said to be triangular if its membership function is given by

$$\mu_{\tilde{M}}(x) = \begin{cases} \dfrac{x-a}{m-a} & a < x \leq m, \\ \dfrac{x-b}{m-b} & m \leq x < b, \\ 0 & \text{otherwise}, \end{cases}$$

where a, m, b are given numbers [10]. Such a fuzzy number $\tilde{M}$ as an interval-valued number is symbolically written

$$\tilde{M} = (a, m, b).$$

Notice that a triangular fuzzy number is not necessarily symmetric, since $m-a$ may be different from $b-m$, however, $\mu_{\tilde{M}}(m) = 1$. Imposing symmetry simplifies the definition of a triangular fuzzy number. Indeed, let m be symmetric in relation to a and b, that is, $m - a = b - m = \delta$. In this case,

$$\mu_{\tilde{M}}(x) = \max\{0, 1 - \frac{|x-m|}{\delta}\}.$$

For α-level set $\tilde{M}_\alpha$, through a linear combination of their lower bound m_α^L, mean value m and upper bound m_α^R, the most likely value $\bar{M}_\alpha$ is defined as

$$\bar{\tilde{M}}_\alpha = \frac{m_\alpha^L + 4m + m_\alpha^R}{6}.$$

The above information about this fuzzy number is shown in Figure 1.

Let $\tilde{M}_1 = (a, m, b)$ and $\tilde{M}_2 = (a', m', b')$ be two triangular fuzzy numbers. The addition $\oplus$ and scalar multiplication $\otimes$ of $\tilde{M}_1$ and $\tilde{M}_2$ are given by the following formulas:

Addition: $\tilde{M}_1 \oplus \tilde{M}_2 = (a + a', m_1 + m', b + b')$

scalar multiplication: $\lambda \otimes \tilde{M} = (\lambda a, \lambda m, \lambda b), \quad \lambda > 0.$

In the model of a two-criteria user-optimized route choice problem, the time and cost functions for transportation are usually described as a mathematical relationship with vague information. To model the imprecision in this problem, the fuzzy set theory is used, in which different type of numbers and the corresponding fuzzy arithmetic on them are necessary for the modeling process.

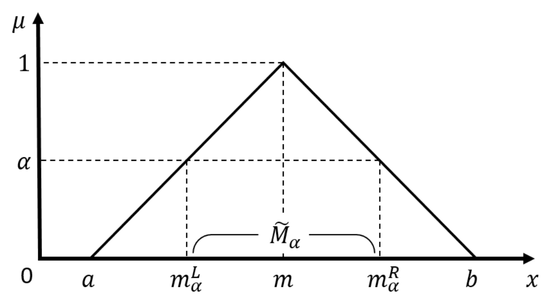

FIGURE 1. α-level set of fuzzy number $\tilde{M}$

3. Fuzzy two-criteria user-optimized route choice model

Let $G(N, A, W)$ be a transportation network, where N denotes the set of nodes, A the set of arcs in the network, and W, the set of O-D nodes. Let $n = |A|$ and $m = |W|$. The sets and symbols used are defined as: a, an arc of the network connecting a pair of nodes, p, a path of the network from an origin to a destination, P_w, the set of paths on $w \in W$, P, the set of network paths i.e. $P = \bigcup_{w \in W} P_w$.

Let f_a denotes the nonnegative traffic flow on arc $a \in A$ and x_p denotes the nonnegative traffic flow on path $p \in P$. A relation between arc flows and path flows is given by

$$f_a = \sum_{p \in P} \delta_{a,p} x_p = \sum_{w \in W} \sum_{p \in P_w} \delta_{a,p} x_p, \tag{3.1}$$

where

$$\delta_{a,p} = \begin{cases} 1 & a \in p, \\ 0 & a \notin p. \end{cases}$$

Let f be the column n-vector of arc flows f_{a_i}, $a_i \in A$, $i = 1, 2, ..., n$ and $x = (x_{p_1}, x_{p_2}, ..., x_{p_m})^T$. We assume that the demand d_w of the traffic flow for each O-D pair $w \in W$ is fixed and

$$d_w = \sum_{p \in P_w} x_p. \tag{3.2}$$

The feasible set Ω consists of all link flows f as

$$\Omega = \{f|\ f \text{ satisfying (3.1) and (3.2)}\}. \tag{3.3}$$

In this paper, two-criteria assumed to be travel time and travel cost. Conceptually, the fuzzy travel time $\tilde{t}_a$ and fuzzy travel cost $\tilde{c}_a$ associated with each link a may be defined as two functions of flows on arc a i.e.,

$$\tilde{t}_a = \tilde{t}_a(f), \quad \tilde{c}_a = \tilde{c}_a(f).$$

With a prespecified membership function and under a certain α-cut level, the fuzzy link travel time function and fuzzy link travel cost function for each link $a \in A$ can be denoted as

$$\tilde{t}_{a,\alpha} = \tilde{t}_{a,\alpha}(f), \quad \tilde{c}_{a,\alpha} = \tilde{c}_{a,\alpha}(f).$$

So, we introduce $\tilde{u}$ as follows:

$$\tilde{u} = \begin{pmatrix} \tilde{t}_{a_1,\alpha} \cdots \tilde{t}_{a_n,\alpha} \\ \tilde{c}_{a_1,\alpha} \cdots \tilde{c}_{a_n,\alpha} \end{pmatrix},$$

then the vector function $\tilde{U}_{p,\alpha}$ as a vector disutility function on path p is computed by

$$\tilde{U}_{p,\alpha} = \bigoplus_{a \in A} \begin{pmatrix} \tilde{t}_{a,\alpha} \\ \tilde{c}_{a,\alpha} \end{pmatrix} \otimes \delta_{a,p}. \tag{3.4}$$

In the classical route choice problem, the Wardrop traffic equilibrium conditions are widely accepted. In a user equilibrium condition associated with the first principle of Wardrop, all the paths that are not used have higher travel costs. Now, we can define the fuzzy vector equilibrium principle, which is the generalization of the Wardrop equilibrium principle and the extension of this crisp equilibrium condition into a fuzzy environment, using the concept of α-cut.

Definition 3.1. (Fuzzy vector equilibrium principle) An arc flow f^* is called a vector equilibrium flow if for all O-D pairs w and for all $p, q \in P_w$,

$$\tilde{U}_{p,\alpha}(f^*) \tilde{\succeq} \tilde{U}_{q,\alpha}(f^*) \Rightarrow x_p = 0, \tag{3.5}$$

where $\tilde{\succeq}$ is a componentwise inequality and denotes the interval-valued number of each component of its left side is *larger than or equal to* the interval-valued number of the corresponding component of its right side but not equal, simultaneously.

4. Main result

We know that operational definitions of *equal* and *larger than or equal to* are hard in practice, so we further simplify the interval-valued travel time $\tilde{t}_{a,\alpha}$ for a triangular membership function by taking their most likely value $\bar{\tilde{t}}_{a,\alpha}$ as follows:

$$\bar{\tilde{t}}_{a,\alpha} = \frac{\tilde{t}^L_{a,\alpha} + 4\tilde{t}^M_a + \tilde{t}^R_{a,\alpha}}{6} \quad \forall a, \alpha, \tag{4.1}$$

where $\tilde{t}^L_{a,\alpha}$, $\tilde{t}^M_a$ and $\tilde{t}^R_{a,\alpha}$ are lower bound, main value and upper bound of $\tilde{t}_p$ in α-cut level set, respectively. The $\bar{\tilde{c}}_{a,\alpha}$ is defined analogously.

As such, equality (3.4) reduces to the following crisp situation:

$$\bar{\tilde{U}}_{p,\alpha} = \sum_{a\in A} \begin{pmatrix} \bar{\tilde{t}}_{a,\alpha} \\ \bar{\tilde{c}}_{a,\alpha} \end{pmatrix} \delta_{a,p}. \tag{4.2}$$

If we define order relations $\geqq$ and $\geq$ for $y, z \in \mathbb{R}^l$ as

$$y \geqq z \Leftrightarrow y_i \geqq z_i, \quad i = 1, 2, ..., l;$$

$$y \geq z \Leftrightarrow y_i \geqq z_i, \quad i = 1, 2, ..., l \quad \text{and } y \neq z,$$

then the equilibrium condition (3.5) reduces also to the following crisp condition:

$$\bar{\tilde{U}}_{p,\alpha}(f^*) \geq \bar{\tilde{U}}_{q,\alpha}(f^*) \Rightarrow x_p = 0. \tag{4.3}$$

We know that $\mathbb{R}^2_+$ is the positive quadrant of $\mathbb{R}^2$, so

$$\bar{\tilde{U}}_{p,\alpha} \geq \bar{\tilde{U}}_{q,\alpha} \Leftrightarrow \bar{\tilde{U}}_{p,\alpha} - \bar{\tilde{U}}_{q,\alpha} \in \mathbb{R}^2_+ \setminus \{0\},$$

and the equilibium condition (4.3) can be defined as follows:

$$\bar{\tilde{U}}_{p,\alpha}(f^*) - \bar{\tilde{U}}_{q,\alpha}(f^*) \in \mathbb{R}^2_+ \setminus \{0\} \Rightarrow x_p = 0. \tag{4.4}$$

In the following theorems, we establish an appropriate relation between a crisp user optimal route choice model and a variational inequality problem.

Theorem 4.1. *At each α-cut level, the arc flow vector f^* is a solution of user optimal condition* (4.4) *if f^* solves the following vector variational inequality problem:*

Find $f^* \in \Omega_\alpha$ such that $\langle \bar{\tilde{u}}(f^*), f - f^* \rangle \notin -\mathbb{R}^2_+ \setminus \{0\},\ \forall f \in \Omega_\alpha, \quad (4.5)$

where the feasible region Ω_α *is delineated by conditions* (3.1)*,* (3.2) *and* (4.2)*.*

Proof. Let f^* satisfies (4.5) and $\bar{\tilde{U}}_{q,\alpha}(f^*) - \bar{\tilde{U}}_{r,\alpha}(f^*) \in \mathbb{R}^2_+ \setminus \{0\}$, where $q, r \in P_w$. We show that $x^*_q = 0$. Choose a path flow vector x as

$$x_p = \begin{cases} x^*_p & p \neq q, r, \\ 0 & p = q, \\ x^*_q + x^*_r & p = r \end{cases}$$

Clearly, x satisfies (3.2) and so the arc flow vector f corresponding to x is in Ω_α. Now,

$$\begin{aligned} \langle \bar{\tilde{u}}(f^*), f - f^* \rangle &= \sum_{a \in A} \begin{pmatrix} \bar{\tilde{t}}_{a,\alpha}(f^*) \\ \bar{\tilde{c}}_{a,\alpha}(f^*) \end{pmatrix} \times (f_a - f^*_a) \\ &= \sum_{w \in W} \sum_{p \in P_w} (\sum_{a \in A} \begin{pmatrix} \bar{\tilde{t}}_{a,\alpha}(f^*) \\ \bar{\tilde{c}}_{a,\alpha}(f^*) \end{pmatrix} \delta_{a,p}) \times (x_p - x^*_p) \\ &= \sum_{w \in W} \sum_{p \in P_w} \bar{\tilde{U}}_{p,\alpha}(f^*) \times (x_p - x^*_p) \\ &= x^*_q (\bar{\tilde{U}}_{r,\alpha}(f^*) - \bar{\tilde{U}}_{q,\alpha}(f^*)) \notin -\mathbb{R}^2_+ \setminus \{0\}. \end{aligned} \tag{4.6}$$

Since

$$\bar{\tilde{U}}_{q,\alpha}(f^*) - \bar{\tilde{U}}_{r,\alpha}(f^*) \in \mathbb{R}^2_+ \setminus \{0\}, \tag{4.7}$$

then (4.6) and (4.7) together imply that $x^*_q = 0$. □

Example 4.2. Consider the network depicted in Figure 2 with a single O-D pair $w = (n_1, n_2)$, in which n_1 is origin and n_2 is destination node. The network consists of two links, $A = \{a, b\}$ and two available paths, $P_w = \{p_1, p_2\}$ with the travel demand $d_w = 30$.

The fuzzy link travel time and fuzzy link travel cost functions are given as follows

$$\begin{aligned} \tilde{t}_a &= f_a \oplus \tilde{2} \otimes f_b, & \tilde{t}_b &= \tilde{3} \otimes f_a \oplus \tilde{6} \otimes f_b, \\ \tilde{c}_a &= \tilde{6} \otimes f_a \oplus \tilde{2} \otimes f_b, & \tilde{c}_b &= \tilde{9} \otimes f_a \oplus \tilde{8} \otimes f_b, \end{aligned}$$

where $\tilde{2} = (1, 2, 3)$, $\tilde{3} = (2, 3, 4)$, $\tilde{6} = (4, 6, 8)$, $\tilde{8} = (6, 8, 10)$ and $\tilde{9} = (7, 9, 11)$. It follows readily that

$$\tilde{u} = \begin{pmatrix} \tilde{t}_a & \tilde{t}_b \\ \tilde{c}_a & \tilde{c}_b \end{pmatrix}, \ \tilde{U}_{p_1} = \begin{pmatrix} \tilde{t}_a \\ \tilde{c}_a \end{pmatrix}, \ \tilde{U}_{p_2} = \begin{pmatrix} \tilde{t}_b \\ \tilde{c}_b \end{pmatrix}.$$

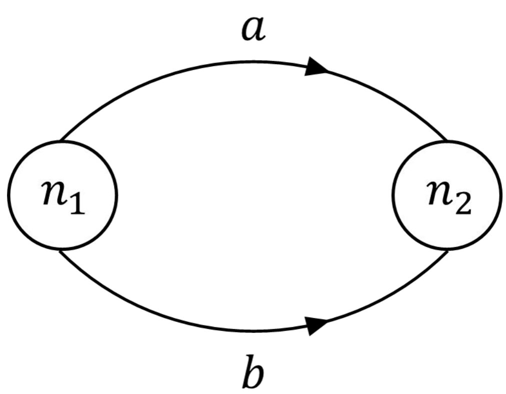

FIGURE 2. A network for the two-criteria example

A fuzzy equilibrium problem for this network is to find an arc flow $f^* = (f_a^* \ f_b^*)^T$ that for $p_1, p_2 \in P_w$

$$\tilde{U}_{p_i,\alpha}(f^*)\tilde{\succeq}\tilde{U}_{p_j,\alpha}(f^*) \Rightarrow x_{p_i} = 0, \quad i,j = 1,2, i \neq j. \tag{4.8}$$

When the $\alpha - cut$ level is set to zero, the above fuzzy link travel time and travel cost functions induce the following most likely link travel time and most likely link travel cost functions:

$$\bar{\tilde{t}}_a = f_a + 2f_b, \qquad \bar{\tilde{t}}_b = 3f_a + 6f_b$$
$$\bar{\tilde{c}}_a = 6f_a + 2f_b, \qquad \bar{\tilde{c}}_b = 9f_a + 8f_b.$$

So,

$$\bar{\tilde{u}} = \begin{pmatrix} \bar{\tilde{t}}_a & \bar{\tilde{t}}_b \\ \bar{\tilde{c}}_a & \bar{\tilde{c}}_b \end{pmatrix}, \ \tilde{U}_{p_1} = \begin{pmatrix} \bar{\tilde{t}}_a \\ \bar{\tilde{c}}_a \end{pmatrix}, \ \bar{\tilde{U}}_{p_2} = \begin{pmatrix} \bar{\tilde{t}}_b \\ \bar{\tilde{c}}_b \end{pmatrix}.$$

The equilibrium problem (4.4) for (4.8) is

$$\bar{\tilde{U}}_{p_i}(f^*) - \bar{\tilde{U}}_{p_j}(f^*) \in \mathbb{R}^2_+ \setminus \{0\} \Rightarrow x_{p_i} = 0, \quad i,j = 1,2, i \neq j. \tag{4.9}$$

The variational inequality (4.5) for this problem is

$$\text{Find } f^* \in \Omega \text{ such that } \langle \bar{\tilde{u}}(f^*), f - f^* \rangle \notin -\mathbb{R}^2_+ \setminus \{0\}, \ \forall f \in \Omega, \tag{4.10}$$

where

$$\Omega = \{f \in \mathbb{R}^2 | x_{p_1} + x_{p_2} = 30, f_a = x_{p_1} \geq 0, f_b = x_{p_2} \geq 0\}.$$

Since $\bar{\tilde{u}}$ is monotone and Lipschitz continuous on Ω, solving the variational inequality (4.10) yields the solution $f^* = (30 \ 0)^T$. It is easy to see that $f^* = (30 \ 0)^T$ is an equilibrium flow for (4.9). Therefore, the result of Theorem 4.1 holds.

Let $\Lambda^{>0} = \left\{\lambda = (\lambda_1, \lambda_2) > 0 \mid \sum_{i=1}^{2} \lambda_i = 1\right\}$. We now show that under some restrictive conditions, the converse result holds.

Theorem 4.3. *At each α-cut level, let λ_1 and λ_2 denote the weights associated with travel time and travel cost, respectively. Assume that arc flow vector f^* is a vector equilibrium flow. Also assume, there exists $\lambda \in \Lambda^{>0}$ so that for each $w \in W$, there is only positive flow on those paths q that their weighted disutility function $\lambda\bar{\tilde{U}}_{q,\alpha}(f^*)$ is minimal. Then f^* solves the vector variational inequality problem* (4.5).

Proof. By the assumptions of the theorem we have

$$\begin{aligned}&\text{for all } w \in W \text{ and for all } p, q \in P_w\\ &\lambda\bar{\tilde{U}}_{p,\alpha}(f^*) \geq \lambda\bar{\tilde{U}}_{q,\alpha}(f^*) \Rightarrow x_p^* = 0,\end{aligned} \tag{4.11}$$

therefore, the single objective version of the equilibrium condition (4.3) is satisfied.

Now, we show that the feasible solution f^* of (4.11) does also satisfy the following equilibrium condition proposed by Nagurney [11]

$$\lambda\bar{\tilde{U}}_{q,\alpha}(f^*) \begin{cases} = \pi_w & \text{if } x_q > 0, \\ \geq \pi_w & \text{if } x_q = 0, \end{cases} \tag{4.12}$$

where π_w is an indicator, whose value is not known a priori.

Suppose that f^* is an equilibrium pattern flow satisfying (4.11). In equilibrium condition (4.12), let $x_q > 0$ for $q \in P_w$. If $\lambda\bar{\tilde{U}}_{q,\alpha}(f^*) - \pi_w \neq 0$, we get $\lambda\bar{\tilde{U}}_{q,\alpha}(f^*) > \pi_w$. Because π_w is the minimum cost function of the set $\{\lambda\bar{\tilde{U}}_{q,\alpha} | q \in P_w\}$ and $|P_w|$ is finite, the set $\{\lambda\bar{\tilde{U}}_{q,\alpha} | q \in P_w\} \subset R_+$ reaches its minimum at $q' \in P_w$ i.e. $\lambda\bar{\tilde{U}}_{q',\alpha}(f^*) = \pi_w$. Thus, $\lambda\bar{\tilde{U}}_{q,\alpha}(f^*) \geq \lambda\bar{\tilde{U}}_{q',\alpha}(f^*)$ and by (4.11) we have $x_q = 0$. This is a contradiction. Therefore, $\lambda\bar{\tilde{U}}_{q,\alpha}(f^*) = \pi_w$ if $x_q > 0$. Obviously, it holds that $\lambda\bar{\tilde{U}}_{q,\alpha}(f^*) \geq \pi_w$ whenever $x_q = 0$. So, f^* satisfies the equilibrium condition (4.12).

Additionally, similar to the proof of Theorem 1 in [11] we find that (4.12) is equivalent to the following variational inequality

$$(VI_\lambda) \quad \text{Find } f^* \in \Omega_\alpha \text{ such that } \langle \lambda\bar{\tilde{u}}(f^*), f - f^* \rangle \geqq 0, \ \forall f \in \Omega_\alpha, \tag{4.13}$$

which is called *scalarized variational inequality* VI_λ. Since f^* satisfies VI_λ with $\lambda \in \Lambda^{>0}$,

$$0 \leqq \langle \lambda\bar{\tilde{u}}(f^*), f - f^* \rangle = \lambda\langle \bar{\tilde{u}}(f^*), f - f^* \rangle, \ \forall f \in \Omega_\alpha,$$

then there cannot exist $f \in \Omega_\alpha$ such that $\langle \bar{\tilde{u}}(f^*), f - f^* \rangle \in -\mathbb{R}_+^2 \setminus \{0\}$. This means that f^* is a solution of $VVIP$ (4.5). □

5. Conclusion

With respect to the uncertainty as a crucial issue in the user-optimized route choice problem, a two-criteria user-optimized route choice model with fuzzy travel time and fuzzy travel cost is induced in this paper. We utilize interval-valued numbers to exhibit the imprecise values of travel time and travel cost on each link of the network. Such numbers reflect the traveler's perception of travel time and travel cost. Then, we define the fuzzy vector equilibrium principle, which is the generalization and extension of the Wardrop equilibrium principle in the static user-optimized route choice problem. However, this definition involves interval arithmetical operations at each α-cut level, which is prohibitive for practical transportation problems. We further simplify the fuzzy equilibrium condition taking the most likely estimation of each link travel time and travel cost that is associated with fuzzy link travel time and fuzzy link travel cost at an α-cut level. As such, a crisp vector equilibrium principle results. In view of this simplification, a vector variational inequality problem is introduced and the relation between the solution of this problem and the traffic equilibrium flow is established.

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