Caspian Journal of Mathematical Sciences (CJMS)
University of Mazandaran, Iran
http://cjms.journals.umz.ac.ir
ISSN: 2676-7260
CJMS. **11**(1)(2022), 170-180

Fuzzy decisive set method for solving multiobjective linear programming problem with intuitionistic fuzzy parameters as a new approach

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Abstract. In this article, we construct a new computational algorithm for solving multiobjective linear programming problem in intuitionistic fuzzy environment. The resources and technological coefficients are taken to be intuitionistic fuzzy numbers. Here, the intuitionistic fuzzy multi-objective linear programming problem is transformed into an equivalent crisp multi-objective linear programming problem. By using fuzzy mathematical programming approach, the transformed multiobjective linear programming problem is reduced into a single objective nonlinear and non-convex programming problem. Stepwise algorithm is given for solving an intuitionistic fuzzy multiobjective linear programming problem and it is checked with a numerical example using intuitionistic fuzzy decisive set method.

Keywords: Intuitionistic fuzzy set, Multi-objective linear programming problem, Membership and non-membership functions, Intuitionistic index, fuzzy decisive set method.

2020 Mathematics subject classification: 90C70, 90C29, 03B20.

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Received: 27 April 2020
Revised: 23 March 2021
Accepted: 31 March 2021

1. Introduction

Linear programming is a powerful optimization technique which generally occurs in in many areas of engineering and management. Since real world problems are very complex, experts and decision makers (DMs) frequently do not know the values of parameters precisely. So, considering the uncertainty, characterizing basic parameters of the model might be more applicable. Since fuzzy Set (FS) becomes famous tool to capture uncertainty and vagueness in real life decision-making problems, therefore the fuzzy linear programming problems (FLPPs) with fuzzy parameters would be viewed as more effective than the conventional one in solving real physical problems. Even, in most of the cases of judgements, evaluation is done by human beings, i.e., by DMs where certainly there are limitations on availabilities and exactness of data. Naturally, every DM hesitates more or less on every evaluation activity. This gives the concept of intuitionistic fuzzy set (IFS) theory which is introduced by Atanassov [1]. The major advantage of IFS over fuzzy set is that IFS separates the degree of acceptance and the degree of non-acceptance of a decision. The IFS theory is generalization of fuzzy theory, so any method for IFS theory is automatically applicable in fuzzy theory as a particular case. So, developing a method for IFS theory is more applicable than for ordinary fuzzy set theory and that is our intention for writing this paper. Applications of these sets have been broadly studied in other aspects such as image processing [6], multi-criteria decision making [16], pattern recognition [15], Medical diagnosis [17], etc.

Several researchers have studied IFS and presented optimization methods such as Angelov [2] has broadened the fuzzy optimization into IF optimization. Basic arithmetic operations of TIFNs is defined by Deng-Feng Li in [6] using membership and non-membership values. Since this fuzzy set generalization can present the degrees of membership and non-membership of an element of the set with a degree of hesitancy, the knowledge and semantic representation becomes more meaningful and applicable.

Dubey and Mehra [8] and Dubey et al. [7] considered the fuzzy linear programming under interval uncertainty based on intuitionistic fuzzy set representation. Li [11] studied interval-valued intuitionistic fuzzy (IVIF) sets and applied it multiattribute decision-making problem. Recently, Bharati and Malhotra [4, 13] have studied intuitionistic fuzzy and its applications in two-stage time minimizing transportation problem. Mondal and Samanta [14] generalized IF sets and presented the concept of generalized intuitionistic fuzzy sets. Ye [19] discussed expected value method for intuitionistic trapezoidal fuzzy multicriteria decision-making problems. Wan and Dong [18] used possibility degree method for interval-valued intuitionistic fuzzy numbers for decision making. Kabiraj et.al., [10] proposed a general tool for modelling problems of decision making under uncertainty where, the degree of rejection is defined simultaneously with the degree of acceptance of a piece of information in such a way that these degrees are not complement to each other. Bharati et.al., [5] introduced a computational algorithm for the solution of MOLP problem in interval-valued intuitionistic fuzzy environment.

In this paper, we consider the IFMOLP problem when technological coefficient and resources are IF numbers. First the given IFMOLP problem is transformed into a deterministic MOLP problem. Using Bellman and Zadeh's fuzzy decision-making process, the MOLP problem is converted into an equivalent crisp nonlinear programming problem. The nonlinear programming problem is solved by fuzzy decisive set method.

The remainder of this paper is organized as follows. In Section 2, the basic concepts which are utilized in this paper will be expressed. A general model of multiobjective fuzzy linear programming problem with intuitionistic fuzzy resources and the technological coefficients is presented in Section 3. Then, a method for solving IFMOLP problem using Fuzzy Mathematical Programming approach is developed. In Section 4, an applicative example is given to verify the proposed approach. Finally, the paper is concluded in Section 5.

2. Preliminaries

In this section, some basic definitions and properties of triangular intuitionistic fuzzy numbers (TIFNs) are presented.

Definition 2.1. [1] An intuitionistic fuzzy set (IFS) $\tilde{A}$ assigns to each element x of the universe X a membership degree $\mu_{\tilde{A}}(x) \in [0,1]$ and a non-membership degree $\nu_{\tilde{A}}(x) \in [0,1]$ such that $\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$. An IFS $\tilde{A}$ is mathematically represented as $\{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle | x \in X\}$.

Definition 2.2. [1] For every common fuzzy subset $\tilde{A}$ on X, Intuitionistic Fuzzy Index of x in $\tilde{A}$ is defined as $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)$. It is also known as degree of hesitancy or degree of uncertainty of the element x in $\tilde{A}$. Obviously, for every $x \in X$, $0 \leq \pi_{\tilde{A}}(x) \leq 1$.

Definition 2.3. [1] An intuitionistic fuzzy set $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle | x \in X\}$ is said to be intuitionistic fuzzy normal if there exist at least two points $x_0, x_1 \in X$ such that $\mu_{\tilde{A}}(x_0) = 1,\ \nu_{\tilde{A}}(x_1) = 1$.

Definition 2.4. [12] An intuitionistic fuzzy set $\tilde{A}$ is said to be intuitionistic fuzzy number, if it is

i) Intuitionistic fuzzy normal.

ii) Convex for the membership function $\mu_{\tilde{A}}(x)$, i.e., $\mu_{\tilde{A}}(\lambda x_1+(1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$ for every $x_1, x_2 \in \mathbb{R},\ \lambda \in [0,1]$.
iii) Concave for the non-membership function $\nu_{\tilde{A}}(x)$, i.e., $\nu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq \max(\nu_{\tilde{A}}(x_1), \nu_{\tilde{A}}(x_2))$ for every $x_1, x_2 \in \mathbb{R},\ \lambda \in [0,1]$.

Definition 2.5. A triangular intuitionistic fuzzy number (TIFN) $\tilde{A}$ is an IFS in $\mathbb{R}$ with membership function and non-membership function as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-(a-\alpha)}{\alpha} & x \in [a-\alpha, a], \\ \frac{\alpha+\beta-x}{\beta} & x \in [a, a+\beta], \\ 0 & otherwise. \end{cases} \quad \nu_{\tilde{A}}(x) = \begin{cases} \frac{a-x}{\alpha'} & x \in [a-\alpha', a], \\ \frac{x-a}{\beta'} & x \in [a, a+\beta'], \\ 1 & otherwise. \end{cases}$$

where $a \in \mathbb{R},\ \alpha, \beta, \alpha', \beta' \geq 0$ such that $\alpha \leq \alpha'$ and $\beta \leq \beta'$, The symbolic representation of TIFN is $\tilde{A} = [a : \alpha, \beta, \alpha', \beta']$, Here α and β are called left and right spreads of membership function $\mu_{\tilde{A}}(x)$, respectively. α' and β' represent left and right spreads of non-membership function $\nu_{\tilde{A}}(x)$, respectively.

3. Intuitionistic fuzzy multiobjective linear programming problem

Consider an intuitionistic fuzzy multiobjective linear programming problem (IFMOLPP) with intuitionistic fuzzy technological coefficients and right hand said constants as follows:

$$\begin{array}{ll} \max & Z = [Z_1, Z_2, \cdots, Z_k] \\ \text{s.t.} & \sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i, \qquad i = 1, \cdots, m, \\ & \qquad x_j \geq 0, \qquad j = 1, \cdots, n, \end{array} \tag{3.1}$$

where $Z_r = \sum_{j=1}^{n} c_{rj} x_j,\ r = 1, \cdots, k$.
Suppose $\tilde{a}_{ij}$ and $\tilde{b}_i$ be IFNs with membership functions and non-membership functions as follows:

$$\mu_{\tilde{a}_{ij}}(x) = \begin{cases} 1 & x < a_{ij} \\ (a_{ij} + d_{ij} - x)/d_{ij} & a_{ij} \leq x \leq a_{ij} + d_{ij}, \\ 0 & x > a_{ij} + d_{ij}. \end{cases}$$

$$\nu_{\tilde{a}_{ij}}(x) = \begin{cases} 0 & x < a_{ij} \\ 1 - c - \mu_{\tilde{a}_{ij}}(x) & a_{ij} \leq x \leq a_{ij} + d_{ij}, \\ 1 & x > a_{ij} + d_{ij}. \end{cases}$$

$$\mu_{\tilde{b}_i}(x) = \begin{cases} 1 & x < b_i \\ (b_i + p_i - x)/p_i & b_i \leq x \leq b_i + p_i, \\ 0 & x > b_i + p_i. \end{cases}$$

$$\nu_{\tilde{b}_i}(x) = \begin{cases} 0 & x < b_i \\ 1 - c - \mu_{\tilde{b}_i}(x) & b_i \le x \le b_i + p_i, \\ 1 & x > b_i + p_i. \end{cases}$$

Where $x \in \mathbb{R},\ d_{ij}, p_i > 0$ for all $i = 1, \cdots, m,\ j = 1, \cdots, n$, and c is called the intuitionistic fuzzy index and the value of c is chosen such that $0 < c < 1$.

In order to defuzzificate the problem (3.1), firstly, we shall obtain the lower and upper bounds of the optimal values which are referred to as Z_r^L and Z_r^U, respectively. The bounds of the optimal values Z_r^L and Z_r^U are obtained by solving the standard linear programming problems as fallows:

$$\begin{array}{rlll} Z_r^1 = & \max & \sum_{j=1}^n c_{rj}x_j & r = 1, \cdots, k \\ & \text{s.t.} & \sum_{j=1}^n (a_{ij} + d_{ij})x_j \le b_i, & i = 1, \cdots, m, \\ & & x_j \ge 0, & j = 1, \cdots, n, \end{array} \tag{3.2}$$

$$\begin{array}{rlll} Z_r^2 = & \max & \sum_{j=1}^n c_{rj}x_j & r = 1, \cdots, k \\ & \text{s.t.} & \sum_{j=1}^n a_{ij}x_j \le b_i + p_i, & i = 1, \cdots, m, \\ & & x_j \ge 0, & j = 1, \cdots, n, \end{array} \tag{3.3}$$

$$\begin{array}{rlll} Z_r^3 = & \max & \sum_{j=1}^n c_{rj}x_j & r = 1, \cdots, k \\ & \text{s.t.} & \sum_{j=1}^n (a_{ij} + d_{ij})x_j \le b_i + p_i, & i = 1, \cdots, m, \\ & & x_j \ge 0, & j = 1, \cdots, n, \end{array} \tag{3.4}$$

$$\begin{array}{rlll} Z_r^4 = & \max & \sum_{j=1}^n c_{rj}x_j & r = 1, \cdots, k \\ & \text{s.t.} & \sum_{j=1}^n a_{ij}x_j \le b_i, & i = 1, \cdots, m, \\ & & x_j \ge 0, & j = 1, \cdots, n, \end{array} \tag{3.5}$$

In the case that all the above LP problems have the finite optimal values, choosing the technological coefficient from the interval $[a_{ij}, a_{ij} + d_{ij}]$ and the right-hand-side numbers from the interval $[b_i, b_i + p_i]$ guarantees that the value of the objective function $\sum_{j=1}^n c_{rj}x_j$ is in the interval $[Z_r^l Z_r^u]$ where $Z_r^l, = \min\{Z_r^1, Z_r^2, Z_r^3, Z_r^4\}$ and $Z_r^u, = \max\{Z_r^1, Z_r^2, Z_r^3, Z_r^4\}$.

Based on the above arguments, we may define the IF set rth of optimal values $\tilde{G}_r(r = 1, \cdots, k)$ as follows:

$$\mu_{\tilde{G}_r}(x) = \begin{cases} 0 & \sum_{j=1}^n c_{rj}x_j < Z_r^l \\ (\sum_{j=1}^n c_{rj}x_j - Z_r^l)/(Z_r^u - Z_r^l) & Z_r^l \le \sum_{j=1}^n c_{rj}x_j < Z_r^u, \\ 1 & \sum_{j=1}^n c_{rj}x_j \ge Z_r^u. \end{cases} \tag{3.6}$$

$$\nu_{\tilde{G}_r}(x) = \begin{cases} 1 & \sum_{j=1}^{n} c_{rj}x_j < Z_r^l \\ 1 - c - \mu_{\tilde{G}_r}(x) & Z_l \leq \sum_{j=1}^{n} c_{rj}x_j < Z_r^u, \\ 0 & \sum_{j=1}^{n} c_{rj}x_j \geq Z_r^u. \end{cases} \tag{3.7}$$

Also, the IF set of the ith constraint, $\tilde{C}_i$, which is a subset of $\mathbb{R}^n$ is defined by

$$\mu_{\tilde{C}_i}(x) = \begin{cases} 0 & b_i < \sum_{j=1}^{n} a_{ij}x_j \\ \frac{b_i - \sum_{j=1}^{n} a_{ij}x_j}{\sum_{j=1}^{n} d_{ij}x_j + p_i} & \sum_{j=1}^{n} a_{ij}x_j \leq b_i < \sum_{j=1}^{n}(a_{ij} + d_{ij})x_j + p_i, \\ 1 & b_i \geq \sum_{j=1}^{n}(a_{ij} + d_{ij})x_j + p_i. \end{cases} \tag{3.8}$$

$$\nu_{\tilde{C}_i}(x) = \begin{cases} 1 & b_i < \sum_{j=1}^{n} a_{ij}x_j \\ 1 - c - \mu_{\tilde{C}_i}(x) & \sum_{j=1}^{n} a_{ij}x_j \leq b_i < \sum_{j=1}^{n}(a_{ij} + d_{ij})x_j + p_i, \\ 0 & b_i \geq \sum_{j=1}^{n}(a_{ij} + d_{ij})x_j + p_i. \end{cases} \tag{3.9}$$

Note that as z approaches its maximum value, the value of c approaches zero.

When the degree of rejection (non-membership) is defined simultaneously with degree of acceptance (membership) of the objective functions and constraints and when both of these degrees are not complementary to each other, then IF sets can be used as a more general tool for describing uncertainty.

Now, by By using the definition of the fuzzy decision proposed by Bellman and Zadeh [3], we have

$$\mu_{\tilde{D}}(x) = \min\{\min_r(\mu_{G_r}(x)), \min_i(\mu_{C_i}(x)), 1 \leq i \leq m, 1 \leq r \leq k\}$$

$$\nu_{\tilde{D}}(x) = \max\{\max_r(\nu_{G_r}(x)), \max_i(\nu_{C_i}(x)), 1 \leq i \leq m, 1 \leq r \leq k\}$$

where $\mu_{\tilde{D}}(x)$ denotes degree of acceptance and $\mu_{\tilde{D}}(x)$ denotes degree of rejection of intuitionistic fuzzy decision. Here, decision maker wants to maximize range of acceptance and minimize range of rejection. Therefore, an intuitionistic fuzzy optimization (IFO) problem is formulated as follows:

$$\begin{aligned} \max \quad & \mu_{\tilde{D}}(x), \\ \min \quad & \nu_{\tilde{D}}(x), \\ \text{s.t.} \quad & \mu_{\tilde{D}}(x) \geq \nu_{\tilde{D}}(x) \\ & 0 \leq \mu_{\tilde{D}}(x) + \nu_{\tilde{D}}(x) \leq 1, \\ & \mu_{\tilde{D}}(x), \mu_{\tilde{D}}(x) \geq 0, \quad x \geq 0. \end{aligned} \tag{3.10}$$

Suppose that $\alpha = \nu_{\tilde{D}}(x)$ and $\beta = \nu_{\tilde{D}}(x)$. Therefore, the IF optimization problem (3.10) can be restated in the form of

$$\begin{array}{ll} \max & \alpha \\ \min & \beta \\ \text{s.t.} & \mu_{\tilde{G}_r}(x) \geq \alpha, \quad \mu_{\tilde{C}_i}(x) \geq \alpha, \\ & \nu_{\tilde{G}_r}(x) \leq \beta, \quad \nu_{\tilde{C}_i}(x) \leq \beta, \\ & \quad \alpha \geq \beta, \qquad 0 \leq \alpha + \beta \leq 1, \\ & \alpha, \beta \geq 0, \qquad x \geq 0. \end{array} \tag{3.11}$$

Further, problem (3.11) can be transformed into an equivalent linear programming problem by applying intuitionistic fuzzy optimization technique

$$\begin{array}{ll} \max & \qquad \alpha - \beta \\ \text{s.t.} & \mu_{\tilde{G}_r}(x) \geq \alpha, \quad \mu_{\tilde{C}_i}(x) \geq \alpha, \\ & \nu_{\tilde{G}_r}(x) \leq \beta, \quad \nu_{\tilde{C}_i}(x) \leq \beta, \\ & \alpha \geq \beta, \qquad 0 \leq \alpha + \beta \leq 1, \\ & \quad \alpha, \beta \geq 0, \qquad x \geq 0. \end{array} \tag{3.12}$$

By setting the relations (3.6), (3.7), (3.8) and (3.9) in the above IFO model, the problem (3.12) can be written as following form:

$$\begin{array}{ll} \max & \alpha - \beta \\ \text{s.t.} & \alpha(Z_r^u - Z_r^l) - \sum_{j=1}^n c_{rj}x_j + Z_r^l \leq 0, \quad r = 1, \cdots, k \\ & \sum_{j=1}^n (a_{ij} + \alpha d_{ij})x_j + \alpha p_i - b_i \leq 0, \quad i = 1, \cdots, m \\ & Z_r^u - c(Z_r^u - Z_r^l) - \beta(Z_r^u - Z_r^l) - \sum_{j=1}^n c_{rj}x_j \leq 0, \quad r = 1, \cdots, k \\ & \sum_{j=1}^n (a_{ij} + \alpha d_{ij} - cd_{ij} - \beta d_{ij})x_j - b_i + (1 - c - \beta)p_i \leq 0, \quad i = 1, \cdots, m \\ & \alpha \geq \beta, \quad 0 \leq \alpha + \beta \leq 1, \quad \alpha, \beta \geq 0, \quad x \geq 0. \end{array} \tag{3.13}$$

It should be emphasized here that α and β are treated as decisive variables, therefore, the constraints in problem (3.13) which containing the cross product terms αx_j and βx_j are not convex. So the problem is a nonlinear programming problem which can not be solved by using usual simplex methods. Therefore the solution of this problem requires the special approach adopted for solving general non-convex optimization problems.

The algorithm of the intuitionistic fuzzy decisive set method. In this method, a combination of the bisection method and phase one of the simplex method of linear programming problem is used to obtain a feasible solution [9]. This method is based on the idea that, for a fixed value of α and β, the problem (3.13) is converted in to linear programming problem. Obtaining the optimal solution α^* and β^* to the problem (3.13) is equivalent to determining the maximum value of α and the minimum value of β so that the feasible set is non-empty. The algorithm of this method for (3.13) is presented below.

Algorithm

Initialization step:

Set $k = 0$. Let α_0, β_0 in the interval $(0, 1)$ such that $\beta_0 = 1 - c - \alpha_0$ where $c \in (0, 1)$ and the difference between α_0 and β_0 should not approach the value zero and test whether a feasible set satisfying the constraints of the problem (3.13) exists or not using phase one of the simplex method. If a feasible set exists, set $\alpha^* = \alpha_0$ and $\beta^* = \beta_0$. Otherwise, set $\alpha_0^L = \beta_0$, $\beta_0^L = \alpha_0$ and $\alpha_0^R = \alpha_0$, $\beta_0^R = \beta_0$ and go to main step.

Main Step: Set $k = k + 1$ and let $\alpha_k = (\alpha_k^L + \alpha_k^R)/2$ and $\beta_k = (\beta_k^L + \beta_k^R)/2$, then update the values of α_k^L, α_k^R, β_k^L and β_k^R using the bisection method as follows:

(i) If feasible set is non-empty for α_k and β_k, set $\alpha_k^L = \alpha_k$ and α_k^R as it's value in the preceding step. $\beta_k^L = \beta_k$ and β_k^R as it's value in the preceding step.

(ii) If feasible set is empty for α_k and β_k, set $\alpha_k^R = \alpha_k$ and α_k^L as it's value in the preceding step. $\beta_k^R = \beta_k$ and β_k^L as it's value in the preceding step.

If $|\alpha_{k+1} - \alpha_k| < \epsilon$ and $|\beta_{k+1} - \beta_k| < \epsilon$, where $\epsilon > 0$ is a small constant, then stop and output $\alpha^* = \alpha_{k+1}$ and $\beta^* = \beta_{k+1}$.

Consequently, for each α_k and β_k, test whether a feasible set of (3.13) exists or not, using phase one of the simplex method and determine the maximum value α^* and the minimum value β^* satisfying the constraints of (3.13).

4. Numerical Example

Example 4.1. Let us consider the following IFMOLPP

$$\begin{aligned} \max \quad & z_1(x) = 10x_1 + 11x_2 + 15x_3 \\ \max \quad & z_2(x) = 4x_1 + 5x_2 + 9x_3 \\ \text{s.t.} \quad & \tilde{1}x_1 + \tilde{1}x_2 + \tilde{1}x_3 \leq \widetilde{15} \\ & \tilde{7}x_1 + \tilde{5}x_2 + \tilde{3}x_3 \leq \widetilde{80} \\ & \tilde{3}x_1 + \widetilde{4.4}x_2 + \widetilde{10}x_3 \leq \widetilde{100} \\ & x_1, x_2, x_3 \geq 0, \end{aligned} \tag{4.1}$$

which take intuitionistic fuzzy parameters as $\tilde{1} = L(1, 1)$, $\tilde{7} = L(7, 4)$, $\tilde{5} = L(5, 3)$, $\tilde{3} = L(3, 1)$, $\widetilde{4.4} = L(4.4, 2)$, $\widetilde{10} = L(10, 4)$, $\widetilde{15} = L(15, 5)$, $\widetilde{80} = L(80, 40)$ and $\widetilde{100} = L(100, 30)$ as used in [9].

That is $(a_{ij}) = \begin{pmatrix} 1 & 1 & 1 \\ 7 & 5 & 3 \\ 3 & 4.4 & 10 \end{pmatrix}, (d_{ij}) = \begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{pmatrix} \Rightarrow (a_{ij} + d_{ij}) = \begin{pmatrix} 3 & 3 & 3 \\ 11 & 8 & 4 \\ 4 & 6.4 & 14 \end{pmatrix}$ and $(b_i) = \begin{pmatrix} 15 \\ 80 \\ 100 \end{pmatrix}, (p_i) = \begin{pmatrix} 50 \\ 40 \\ 30 \end{pmatrix} \Rightarrow (b_i + p_i) = \begin{pmatrix} 20 \\ 120 \\ 130 \end{pmatrix}$

For solving this problem, firstly, we must obtain the lower and upper bounds of the optimal values which are obtained by solving four sub-problems (3.2)-(3.5). Optimal values of these problems are $Z_1 = (110, 250, 145, 189.29)$ and $Z_2 = (65, 130, 85, 99.29)$ respectively. Therefore, $Z_1^l = 110$, $Z_1^u = 250$, $Z_2^l = 65$ and $Z_2^u = 130$. By using these optimal values, the problem (4.1) can be reduced by the following nonlinear programming problem:

$$
\begin{aligned}
\max\ & \alpha - \beta \\
\text{s.t.}\ & 10x_1 + 11x_2 + 15x_3 \geq 140\alpha + 110, \\
& 4x_1 + 5x_2 + 9x_3 \geq 65\alpha + 65, \\
& (\alpha + 1)x_1 + (\alpha + 1)x_2 + (\alpha + 1)x_3 \leq 15 - 5\alpha, \\
& (4\alpha + 7)x_1 + (3\alpha + 5)x_2 + (\alpha + 3)x_3 \leq 80 - 40\alpha, \\
& (\alpha + 3)x_1 + (2\alpha + 4.4)x_2 + (4\alpha + 10)x_3 \leq 100 - 30\alpha, \\
& 10x_1 + 11x_2 + 15x_3 \geq 250 - 140c - 140\beta, \\
& 4x_1 + 5x_2 + 9x_3 \geq 130 - 65c - 65\beta, \\
& (2 - c - \beta)x_1 + (2 - c - \beta)x_2 + (2 - c - \beta)x_3 \leq 50c + 50\beta - 35, \\
& (11 - 4c - 4\beta)x_1 + (8 - 3c - 3\beta)x_2 + (4 - c - \beta)x_3 \leq 40c + 40\beta + 40, \\
& (4 - c - \beta)x_1 + (6.4 - 2c - 2\beta)x_2 + (14 - 4c - 4\beta)x_3 \leq 30c + 30\beta + 70, \\
& x_1, x_2, x_3 \geq 0, \qquad 0 \leq \alpha, \beta \leq 1.
\end{aligned}
$$

Consequently, we obtain the optimal value of at the fifteenth iteration by using the intuitionistic fuzzy decisive set method and taking $\epsilon = 10^{-4}$ (see Table 1). The optimal solution is $x_1^* = 0$, $x_2^* = 0$, $x_3^* = 8.39$, $z_1 = 125.85$, $z_2 = 75.51$.

This means that the vector (x_1^*, x_2^*, x_3^*) is a solution to the problem (4.1) which has the best membership grade $\alpha^* = 0.1132$ and the least non-membership grade $\beta^* = 0.7868$ with intuitionistic index $c = 0.1$.

5. Conclusion

This research proposed intuitionistic fuzzy multiobjective linear programming problem and a method for its solution has been developed.

TABLE 1. intuitionistic fuzzy decisive set method for Example (4.1)

k	α-value	β-value	feasible set
1	0.8	0.1	empty
2	0.45	0.45	empty
3	0.275	0.625	empty
4	0.1875	0.7125	empty
5	0.14375	0.75625	empty
6	0.121875	0.778125	empty
7	0.1109375	0.7890625	non-empty
8	0.1164062	0.7835937	empty
9	0.1136718	0.7863281	empty
10	0.1123046	0.7876953	non-empty
11	0.1129882	0.7870117	non-empty
12	0.11333	0.7866699	empty
13	0.1131591	0.7868408	non-empty
14	0.1132445	0.7867553	empty
15	0.1132018	0.7867981	non-empty

The IFMOLPP is converted into an equivalent crisp non-linear programming problem using the concept of max-min principle. The resultant non-linear programming problem was solved by intuitionistic fuzzy decisive set method. The discussed method was illustrated through an example. The proposed computational algorithm is easy and more accurate for modeling and decision making of optimization problems with multiple objectives under uncertainty and vagueness. Our proposed computational algorithm can be further developed for nonlinear case also, and it may successfully applicable in various sectors such as aircraft control system design, supply chain management, image segmentation and industrial neural network design.

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