Caspian Journal of Mathematical Sciences (CJMS)
University of Mazandaran, Iran
http://cjms.journals.umz.ac.ir
ISSN: 1735-0611

CJMS. **6**(2)(2017), 99-106

Some points on generalized open sets

Shyamapada Modak [1]
[1] Department of Mathematics
University of Gour Banga
P.O. Mokdumpur, Malda 732 103
West Bengal, India

Abstract. The paper is an attempt to represent a study of limit points, boundary points, exterior points, border, interior points and closure points in the common generalized topological space. This paper takes a look at the possibilities of an extended topological space and it also considers the new characterizations of dense set.

Keywords: Limit point, Boundary point, Border, Exterior point, Dense set.

2000 Mathematics subject classification: 54A20, 54A05.

1. Introduction

There has been a significant body of literature that has dealt with the study of generalization of the topological space and the open sets. In this paper, however, we have attempted to accumulate them in a frame. We have considered a type of sets in the topological space which are empty or contain a nonempty open set. These types of sets may be referred to as λ-open. This is also a generalization of open sets, implying that each open set in a topological space is a λ-open set. It also follows that its converse may not be true in general. This paper suggests that this is an extension of the topological space. Some examples of such λ-open sets are semi-open set [3], α-set [7]. On the other hand, $\psi -$

[1] Corresponding author: spmodak2000@yahoo.co.in
Received: 8 July 2016
Revised: 13 November 2016
Accepted: 13 November 2016

C set [6], ψ-set [2], preopen set [4], semi-preopen set [1], ψ^*-sets [5] cannot be categorized as such sets. The complement of a λ-open is called λ-closed, and for a topological space, each closed set is a λ-closed set. Therefore we arrive at two optimal cases, one being the discrete topological space and another being the indiscrete topological space. In these two topological spaces we do not have an extension due to λ-open sets. Now if we accumulate the λ-open sets of finite complement topological space and countable complement topological space, then we arrive again to the finite complement topological space and countable complement topological space respectively. In fact the reader will be interested to $\lambda(\mathbb{R}) \neq \mathbb{R} \cup \mathbb{R}_l \cup \mathbb{R}_u$, where $\mathbb{R}$, $\mathbb{R}_l$ and $\mathbb{R}_u$ denote the standard topology on real line lower-limit topology on real line and upper-limit topology on real line respectively and $\lambda(\mathbb{R})$ denotes the set of all λ-open sets in $\mathbb{R}$. If A is a subset of a topological space X, then we define $Int_\lambda(A)$ by the union of all λ-open sets contained in A and $Cl_\lambda(A)$ is defined by the intersection of all λ-closed sets containing A. These operators 'Int_λ' and 'Cl_λ' satisfy the following results:

Proposition 1.1. *For subsets A, B of a topological space X, the following statements are true:*

(1) $Int_\lambda(A) \subset A$.
(2) $Int(A) \subset Int_\lambda(A)$, *where $Int(A)$ denotes the interior of A in X.*
(3) *if $A \subset B$, then $Int_\lambda(A) \subset Int_\lambda(B)$.*
(4) $Int_\lambda(Int_\lambda(A)) \subset Int_\lambda(A)$.
(5) $Int_\lambda(A \cup B) = Int_\lambda(A) \cup Int_\lambda(B)$.
(6) *A is λ-open if and only if $Int_\lambda(A) = A$.*
(7) $A \subset Cl_\lambda(A)$.
(8) $Int_\lambda(A) \subset A \subset Cl_\lambda(A)$.
(9) $Cl_\lambda(A) \subset Cl(A)$.
(10) *if $A \subset B$, then $Cl_\lambda(A) \subset Cl_\lambda(B)$.*
(11) $Cl_\lambda(A) \cup Cl_\lambda(B) \subset Cl_\lambda(A \cup B)$.
(12) *if A is closed, then $Cl_\lambda(A) = A$.*
(13) *if $x \in Cl_\lambda(A)$, then $U_x \cap A \neq \emptyset$ for all $U_x \in \lambda(x)$, set of all λ-open sets containing x in X.*
(14) $Int_\lambda(A) = X \setminus Cl_\lambda(X \setminus A)$.

2. λ-closure and λ-interior

Definition 2.1. Let A be a subset of a topological space X. A point $x \in X$ is said to be λ-limit point of A if each λ-open set U containing x, $U \cap (A \setminus \{x\}) \neq \emptyset$. The set of all λ-limit points of A is called the λ-derived set of A and is denoted by $D_\lambda(A)$.

Theorem 2.2. *For subsets A, B of a topological space X, the following statements hold:*

(1) $D_\lambda(A) \subset D(A)$ *($D(A)$ denoted as the set of all limit points of A in X).*
(2) *If $A \subset B$, then $D_\lambda(A) \subset D_\lambda(B)$.*
(3) $D_\lambda(A) \cup D_\lambda(B) = D_\lambda(A \cup B)$ *and* $D_\lambda(A \cap B) \subset D_\lambda(A) \cap D_\lambda(B)$.
(4) $D_\lambda(D_\lambda(A)) \setminus A \subset D_\lambda(A)$.
(5) $D_\lambda(A \cup D_\lambda(A)) \subset A \cup D_\lambda(A)$.

Proof. (1) It is obvious from the fact that $O(X)$ (set of all open sets in X) $\subset \lambda(X)$(set of all λ-open sets in X).

(2) Obvious and hence omitted.

(3) From (2), we have $D_\lambda(A) \cup D_\lambda(B) \subset D_\lambda(A \cup B)$. For reverse inclusion, suppose $x \in D_\lambda(A \cup B)$. Then each λ - open set U containing x, $U \cap (A \setminus \{x\}) \neq \emptyset$ and $U \cap (B \setminus \{x\}) \neq \emptyset$. Thus $U \cap (A \cup B \setminus \{x\}) \neq \emptyset$. Therefore $x \in D_\lambda(A \cup B)$. Hence the result.

Second part: It is obvious from (2).

(4) If $x \in D_\lambda(D_\lambda(A)) \setminus A$ and U is λ - open set containing x, then $U \cap (D_\lambda(A) \setminus \{x\}) \neq \emptyset$. Let $y \in U \cap (D_\lambda(A) \setminus \{x\})$. Then since $y \in D_\lambda(A)$ and $y \in U$, $U \cap (A \setminus \{y\}) \neq \emptyset$. Let $z \in U \cap (A \setminus \{y\})$. Then $z \neq x$ for $z \in A$ and $x \notin A$. Hence $U \cap (A \setminus \{x\}) \neq \emptyset$. Therefore $x \in D_\lambda(A)$.

(5) Let $x \in D_\lambda(A \cup D_\lambda(A))$. If $x \in A$, the result is obvious. So let $x \in D_\lambda(A \cup D_\lambda(A)) \setminus A$, then for λ - open set U containing x, $U \cap (A \cup D_\lambda(A) \setminus \{x\}) \neq \emptyset$. Thus $U \cap (A \setminus \{x\}) \neq \emptyset$ or $U \cap (D_\lambda(A) \setminus \{x\}) \neq \emptyset$. Now it follows similarly from (4) that $U \cap (A \setminus \{x\}) \neq \emptyset$. Hence $x \in D_\lambda(A)$. Therefore, in any case $D_\lambda(A \cup D_\lambda(A)) \subset A \cup D_\lambda(A)$. □

The reverse inclusion of 1 of the above Theorem need not hold in general:

Example 2.3. Let $(\mathbf{R}, \mathbb{R})$ be the standard topological space, where $\mathbf{R}$ denoted set of reals. Suppose $A = \{\frac{1}{n} : n \in \mathbb{Z}^+\} \subset \mathbf{R}$. Then $0 \in D(A)$. Now for any $\epsilon > 0$, $(-\epsilon, 0]$ is λ - open set as $(-\epsilon, 0) \subset (-\epsilon, 0]$. Then $(A \setminus \{0\}) \cap (-\epsilon, 0] = \emptyset$, then $0 \notin D_\lambda(A)$.

Theorem 2.4. *For a subset A of a topological space X:*

(1) $Cl_\lambda(A) \subset D_\lambda(A) \cup A \subset Cl(A)$.
(2) $D_\lambda(A) \subset A$, *if A is closed.*
(3) *If A is closed and λ-closed, then* $Cl_\lambda(A) = A \cup D_\lambda(A) = Cl(A) = A \cup D(A) = A$.

Definition 2.5. Let X be a topological space, then $b_\lambda(A) = A \setminus Int_\lambda(A)$ is said to be that λ-border of A.

Theorem 2.6. *For a subset A of a topological space X, the following statements hold:*

(1) $b_\lambda(A) \subset b(A)$ *where* $b(A)$ *denote the border of* A.
(2) $A = Int_\lambda(A) \cup b_\lambda(A)$.
(3) $Int_\lambda(A) \cap b_\lambda(A) = \emptyset$.
(4) A *is a* λ*-open set if and only if* $b_\lambda(A) = \emptyset$.
(5) $Int_\lambda(b_\lambda(A)) = \emptyset$.
(6) $b_\lambda(b_\lambda(A)) = b_\lambda(A)$.
(7) $b_\lambda(A) = A \cap Cl_\lambda(X \setminus A)$.

Proof. (1) It is obvious from the fact that $Int(A) \subset Int_\lambda(A)$.

(2) It is obvious from the fact that $b_\lambda(A) = A \setminus Int_\lambda(A)$.

(3) It is obvious from the fact $b_\lambda(A) = A \setminus Int_\lambda(A)$.

(4) Suppose A is λ-open. Then $A = Int_\lambda(A)$, so $b_\lambda(A) = \emptyset$. Conversely suppose that $b_\lambda(A) = \emptyset$, then $A \subset Int_\lambda(A)$. Thus A is λ-open.

(5) If possible suppose that $Int_\lambda(b_\lambda(A)) \neq \emptyset$. Let $x \in Int_\lambda(b_\lambda(A))$, then $x \in b_\lambda(A)$. Since $b_\lambda(A) \subset A$, then $x \in Int_\lambda(A)$ (since $x \in Int_\lambda(b_\lambda(A)) \subset Int_\lambda(A)$). Thus we have, $x \in Int_\lambda(A) \cap b_\lambda(A)$ which contradicts (3). Thus $Int_\lambda(b_\lambda(A)) = \emptyset$.

(6) $b_\lambda(b_\lambda(A)) = b_\lambda(A) \setminus Int_\lambda(b_\lambda(A)) = b_\lambda(A)$ (since $Int_\lambda(b_\lambda(A)) = \emptyset$).

(7) $b_\lambda(A) = A \setminus Int_\lambda(A) = A \cap (X \setminus Int_\lambda(A)) = A \cap Cl_\lambda(X \setminus A)$. □

Reverse inclusion of (1) of the above Theorem need not hold in general:

Let $(\mathbf{R}, \mathbb{R})$ be the standard topological space. Then for any $\epsilon > 0$, $Int_\lambda([-\epsilon, \epsilon]) \supset Int([-\epsilon, \epsilon])$. Thus $\epsilon \in Int_\lambda([-\epsilon, \epsilon])$ but $\epsilon \notin Int([-\epsilon, \epsilon])$.

Definition 2.7. Let A be a subset of a topological space X, $bd_\lambda(A) = Cl_\lambda(A) \cap Cl_\lambda(X - A)$ is said to λ-boundary of A.

Theorem 2.8. *Let* A *be a subset of a topological space* X*, the following statements hold:*

(1) $bd_\lambda(A) \subset bd(A)$ *(*$bd(A)$ *is denoted as the set of all boundary points of* A *in* X*).*
(2) $Cl_\lambda(A) = Int_\lambda(A) \cup bd_\lambda(A)$.
(3) $b_\lambda(A) \subset bd_\lambda(A)$.
(4) $bd_\lambda(A) = bd_\lambda(X \setminus A)$.
(5) $X \setminus bd_\lambda(A) = Int_\lambda(A) \cup Int_\lambda(X \setminus A)$.
(6) $bd_\lambda(A) = Cl_\lambda(A) \setminus Int_\lambda(A) = Cl_\lambda(X \setminus A) \setminus Int_\lambda(X \setminus A)$.
(7) $bd_\lambda(A)$ *is the set of all* $x \in X$ *such that* $x \notin Int_\lambda(A)$ *and* $x \notin Int_\lambda(X \setminus A)$.
(8) $A \cup bd_\lambda(A) \subset Cl_\lambda(A) \subset Cl(A)$.
(9) $Int_\lambda(A) \subset A \setminus bd_\lambda(A) \subset Cl_\lambda(A)$.
(10) A *is* λ*-closed when* $bd_\lambda(A) \subset A$.
(11) A *is* λ*-open if and only if* $A \cap bd_\lambda(A) = \emptyset$.

Proof. (1) Let $x \in bd_\lambda(A)$. Then $x \in Cl_\lambda(A) \cap Cl_\lambda(X \setminus A) \subset Cl(A) \cap Cl(X \setminus A) = bd(A)$. This implies that $bd_\lambda(A) \subset bd(A)$.

(2) $bd_\lambda(A) \cup Int_\lambda(A) = (Cl_\lambda(A) \cap Cl_\lambda(X \setminus A)) \cup (Int_\lambda(A)) = (Cl_\lambda(A) \cap (X \setminus Int_\lambda(A)) \cup (Int_\lambda(A)) = Cl_\lambda(A)$.

(3) We know $A \setminus Int_\lambda(A) \subset X \setminus Int_\lambda(A)$ and $A \setminus Int_\lambda(A) \subset Cl_\lambda(A)$. Thus $A \setminus Int_\lambda(A) \subset Cl_\lambda(A) \cap (X \setminus Int_\lambda(A)) = Cl_\lambda(A) \cap Cl_\lambda(X \setminus A) = bd_\lambda(A)$. Therefore $b_\lambda(A) \subset bd_\lambda(A)$.

(4) Given that $bd_\lambda(A) = Cl_\lambda(A) \cap Cl_\lambda(X \setminus A) = Cl_\lambda(X \setminus A) \cap Cl_\lambda(X \setminus (X \setminus A)) = bd_\lambda(X \setminus A)$. Thus $bd_\lambda(A) = bd_\lambda(X \setminus A)$.

(5) $X \setminus bd_\lambda(A) = X \setminus (Cl_\lambda(A) \cap Cl_\lambda(X \setminus A)) = (X \setminus Cl_\lambda(A)) \cup (X \setminus Cl_\lambda(X \setminus A)) = Int_\lambda(X \setminus A) \cup Int_\lambda(A)$.

(6) $bd_\lambda(A) = Cl_\lambda(A) \cap Cl_\lambda(X \setminus A) = Cl_\lambda(A) \cap (X \setminus Int_\lambda(A)) = Cl_\lambda(A) \setminus Int_\lambda(A)$.

Second part: We know $bd_\lambda(A) = bd_\lambda(X \setminus A)$. Then we replaced A by $X \setminus A$ in the above relation and we get $bd_\lambda(A) = bd_\lambda(X \setminus A) = Cl_\lambda(X \setminus A) \setminus Int_\lambda(X \setminus A)$.

(7) $bd_\lambda(A) = Cl_\lambda(A) \cap Cl_\lambda(X \setminus A) = Cl_\lambda(A) \cap (X \setminus Int_\lambda(A)) = (X \setminus Int_\lambda(X \setminus A)) \cap (X \setminus Int_\lambda(A))$. Then for $x \in bd_\lambda(A)$, $x \in (X \setminus Int_\lambda(X \setminus A))$ and $x \in (X \setminus Int_\lambda(A))$. Thus $x \notin Int_\lambda(X \setminus A)$ and $x \notin Int_\lambda(A)$. Therefore $bd_\lambda(A)$ is the set of all $x \in X$ such that $x \notin Int_\lambda(A)$ and $x \notin Int_\lambda(X \setminus A)$.

(8) Since $bd_\lambda(A) \subset Cl_\lambda(A)$, then $A \cup bd_\lambda(A) \subset Cl_\lambda(A)$ ($\because Cl_\lambda(A) \cup A = Cl_\lambda(A)$).

(9) Given that $Cl_\lambda(A) \supset A \cup bd_\lambda(A)$. Then $Cl_\lambda(X \setminus A) \supset (X \setminus A) \cup bd_\lambda(X \setminus A) = (X \setminus A) \cup bd_\lambda(A)$. Thus $X \setminus Int_\lambda(A) \supset (X \setminus A) \cup bd_\lambda(A)$, and $Int_\lambda(A) \subset (X \setminus (X \setminus A)) \cap (X \setminus bd_\lambda(A)) = A \cap (X \setminus bd_\lambda(A)) = A \setminus bd_\lambda(A)$.

(10) We have $bd_\lambda(A) \cup A \subset Cl_\lambda(A) = A$ as A is λ-closed. Thus $bd_\lambda(A) \cup A \subset A$, and hence $bd_\lambda(A) \subset A$.

(11) Suppose that $A \cap bd_\lambda(A) = \emptyset$. Then $A \cap (Cl_\lambda(A) \cap Cl_\lambda(X \setminus A)) = \emptyset$, and $A \cap Cl_\lambda(X \setminus A) = \emptyset$. This implies that $A \cap (X \setminus Int_\lambda(A)) = \emptyset$, and hence $A \setminus Int_\lambda(A) = \emptyset$. So $A \subseteq Int_\lambda(A)$, and $A = Int_\lambda(A)$. Therefore A is λ-open.

Suppose A is λ-open in X. If possible $A \cap bd_\lambda(A) \neq \emptyset$. Suppose $x \in A \cap bd_\lambda(A)$, then $x \in A$ and $x \in Cl_\lambda(A) \cap Cl_\lambda(X \setminus A)$. Then $x \in X \setminus A$ as A is λ-open set in X, a contradiction to the fact that $x \in A$. Thus $A \cap bd_\lambda(A) = \emptyset$. □

The reverse inclusion of (1) of the above Theorem need not hold in general.

Example 2.9. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Then $\lambda(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $C(\lambda(X))$ (all λ-closed sets in X) $= \{\emptyset, X, \{b, c\}, \{a, c\}, \{c\}, \{b\}, \{a\}\}$. Let $A = \{b\}$, then $bd_\lambda(A) = Cl_\lambda(\{b\}) \cap Cl_\lambda(\{a, c\}) = \emptyset$ but $bd(A) = Cl(\{b\}) \cap Cl(\{a, c\}) \neq \emptyset$.

Definition 2.10. $Ext_\lambda(A) = Int_\lambda(X \setminus A)$ is said to be a λ-exterior of A.

Theorem 2.11. *For a subset A of a topological space X, the following statements hold:*

(1) $Ext(A) \subset Ext_\lambda(A)$ *where $Ext(A)$ denotes the exterior of A.*
(2) $Ext_\lambda(A)$ *is open.*
(3) $Ext_\lambda(A) = Int_\lambda(X \setminus A) = X \setminus Cl_\lambda(A)$.
(4) $Ext_\lambda(Ext_\lambda(A)) = Int_\lambda(Cl_\lambda(A))$.
(5) *If $A \subset B$, then $Ext_\lambda(A) \supset Ext_\lambda(B)$.*
(6) $Ext_\lambda(A \cup B) \subset Ext_\lambda(A) \cup Ext_\lambda(B)$.
(7) $Ext_\lambda(A \cap B) \supset Ext_\lambda(A) \cap Ext_\lambda(B)$.
(8) $Ext_\lambda(X) = \emptyset$.
(9) $Ext_\lambda(\emptyset) = X$.
(10) $Ext_\lambda(X \setminus Ext_\lambda(A)) \subset Ext_\lambda(A)$.
(11) $Int_\lambda(A) \subset Ext_\lambda(Ext_\lambda(A))$.
(12) $X = Int_\lambda(A) \cup Ext_\lambda(A) \cup bd_\lambda(A)$.

Proof. (1) Obvious and hence omitted.

(2) It is obvious from $Ext_\lambda(A) = Int_\lambda(A) = Int_\lambda(X \setminus A)$.

(3) It is obvious from $Ext_\lambda(A) = Int_\lambda(A) = Int_\lambda(X \setminus A)$.

(4) $Ext_\lambda(Ext_\lambda(A)) = Ext_\lambda(X \setminus Cl_\lambda(A)) = Int_\lambda(X \setminus (X \setminus Cl_\lambda(A))) = Int_\lambda(Cl_\lambda(A))$.

(5) Since $(X \setminus A) \supset (X \setminus B)$ as $A \subset B$. Then $Int_\lambda(X \setminus A) \supset Int_\lambda(X \setminus B)$. Hence the result.

(6) We have from (5), $Ext_\lambda(A \cup B) \subset Ext_\lambda(A) \cup Ext_\lambda(B)$.

(7) It is obvious from (5).

(8) It is obvious from the relation $Ext_\lambda(A) = Int_\lambda(X \setminus A)$.

(9) It is obvious from the fact that $Ext_\lambda(A) = Int_\lambda(X \setminus A)$.

(10) $Ext_\lambda(X \setminus Ext_\lambda(A)) = Ext_\lambda(X \setminus Int_\lambda(X \setminus A)) = Int_\lambda(X \setminus (X \setminus Int_\lambda(X \setminus A))) = Int_\lambda(Int_\lambda(X \setminus A)) \subset Int_\lambda(X \setminus A) = Ext_\lambda(A)$.

(11) $Int_\lambda(A) \subset Int_\lambda(Cl_\lambda(A)) = Int_\lambda(X \setminus Int_\lambda(X \setminus A)) = Int_\lambda(X \setminus Ext_\lambda(A)) = Ext_\lambda(Ext_\lambda(A))$.

(12) We have $Int_\lambda(A) \cup Ext_\lambda(A) \cup bd_\lambda(A) = Int_\lambda(A) \cup (Cl_\lambda(A) \cap Cl_\lambda(X \setminus A)) \cup Ext_\lambda(A) = Cl_\lambda(A) \cup Ext_\lambda(A) = Cl_\lambda(A) \cup Int_\lambda(X \setminus A) = Cl_\lambda(A) \cup (X \setminus Cl_\lambda(A)) = X$. □

The relation '$\subset$' can not be replaced by '$=$' of $Ext_\lambda(A \cup B) \subset Ext_\lambda(A) \cup Ext_\lambda(B)$, which is followed by the following Example:

Example 2.12. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, then $\lambda(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Suppose $A = \{a, b\}$ and $B = \{b, c\}$, then $Ext_\lambda(A) = \emptyset$ and $Ext_\lambda(B) = \{a\}$ but $Ext_\lambda(A \cup B) = \emptyset$. Thus $Ext_\lambda(A \cup B) \neq Ext_\lambda(A) \cup Ext_\lambda(B)$.

Theorem 2.13. *Let A be a subset of a topological space X. Then following holds:*

1. *A is dense in X if and only if $A \cap U \neq \emptyset$ for every nonempty λ-open set U in X.*
2. *If $Int_\lambda(A) = \emptyset$, then $X \setminus A$ is dense in X.*
3. *$X \setminus b_\lambda(A)$ is dense in X.*
4. *If $Ext_\lambda(A) = \emptyset$, then A is dense in X.*

Proof. (1) Suppose A is dense in X. Then for every nonempty open subset V of X, $V \cap A \neq \emptyset$. Let U be a nonempty λ-open set in X. Since U contains a nonempty open subset of X, then $U \cap A \neq \emptyset$.

Conversely suppose that $A \cap U \neq \emptyset$ for every nonempty λ-open sets U in X. As every open sets is also a λ-open set, thus $A \cap V \neq \emptyset$ for every nonempty open set V of X.

(2) $Cl_\lambda(X \setminus A) = X \setminus Int_\lambda(A) = X$ (as $Int_\lambda(A) = \emptyset$), thus $X \setminus A$ is dense in X.

(3) Since $Int_\lambda(b_\lambda(A)) = \emptyset$ (from Theorem 2.6(5)), then $X \setminus b_\lambda(A)$ is dense in X.

(4) Let $Ext_\lambda(A) = \emptyset$. Then from Theorem 2.11(12), $X = Int_\lambda(A) \cup bd_\lambda(A) = Cl_\lambda(A)$ (from Theorem 2.8(2)) □

3. **Conclusion**

In this paper, we have discussed the idea of limit points, border, boundary points, exterior points etc. We have considered extensions of the topological space with help of a new type of sets which is the generalization of generalized open sets in the topological spaces.

Acknowledgement: (i) The author is thankful to the referees for their valuable suggestions.

(ii) The author is also acknowledging Dr. Samipendra Banerjee for his suggestions regarding language.

References

[1] D. Andrijević, On the topology generated by preopen sets, *Mathemathhkh Bechhk.* **39** *(1987), 463-466.*

[2] C. Bandyopadhyaya, and S. Modak, A new topology via ψ - operator, *Proc. Natl. Acad. Sci., India, Sect. A Phys. Sci.* **76** *(2006), 17-20.*

[3] N. Levine, Semi - open sets and semi - continuity in topological spaces, *Amer. Math. Monthly.* **70** *(1963), 36-41.*

[4] A. S. Mashhour, M. E. Abd. El-Monsef, and S. N. El-Deeb, On precontinuous and week precontinuous mappings, *Proc. Math. Phys. Soc. Egypt.* **53** *(1982), 47-53.*

[5] S. Modak, Some new topologies on ideal topological spaces, *Proc. Natl. Acad. Sci., India, Sect. A Phys. Sci.* **82**(**3**) *(2012), 233-243.*

[6] S. Modak, and C. Bandyopadhyaya, A note on ψ - operator, *Bull. Malyas. Math. Sci. Soc.* **30**(**1**) *(2007), 43-48.*

[7] O. Njastad, On some classes of nearly open sets, *Pacific J. Math.* **15** *(1965), 961-970.*